A Generalized Convergence Criterion to Achieve Maximum Fairness Among Users in Downlink Asynchronous Networks Using OFDM/FBMC

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Abstract—In this letter, we address the problem of maximizing the minimum user rates known as rate adaptive (RA) for asynchronous cellular networks with multicarrier modulation such as filter-bank-based multicarrier and orthogonal frequency-division multiplexing. By exploiting the relationship between margin adaptive (MA) and RA, we propose a general convergence criterion to solve the MA power control problem permitting to achieve maximum fairness among all users for the RA problem. Simulation results demonstrate a remarkable improvement in terms of higher minimum user data rates for the RA when the proposed convergence criterion is utilized over the traditional iterative waterfilling for the power control scheme of the MA problem.

Index Terms—OFDM, FBMC, asynchronous networks, inter-cell interference management, resource allocation and power control.

I. INTRODUCTION

THE performance of systems using orthogonal frequency division multiplexing (OFDM) modulation highly depends on synchronicity. For future 5G cellular networks, where the synchronicity constraint is likely to be relaxed [1], filter bank based multi-carrier (FBMC) modulation seems to be a more appealing modulation scheme. In this letter, we focus on a downlink cellular network where base stations (BS) are randomly deployed without any inter-connection or coordination leading to an asynchronous transmission among them. This scenario corresponds, for instance, to professional mobile radio (PMR) for disaster relief where existing networks have been destroyed and a temporary network is needed.

Similar setting was investigated in [2], where the authors studied the distributed sum rate maximization problem. Different from the work in [2], we consider the rate adaptive (RA) problem which consists in maximizing the minimum rate subject to a total power constraint. This optimization problem was investigated in [3] and [4] either for the case of synchronized orthogonal frequency division multiple access (OFDMA) or multi-user OFDM. In [4], Kim et al. built the connection between RA problem and the margin adaptive (MA) optimization which minimizes the total transmit power subject to a rate constraint for users. MA optimization has been solved for OFDMA in the case of perfect synchronization and low level of coordination between BSs in [5] and [6], respectively. To the best of our knowledge, no solution has yet been investigated in the case of asynchronous transmission among the BSs. In this letter, we extend the scheme in [5] to asynchronous networks and propose a sufficient decentralized criterion to be met to ensure convergence of iterative power control algorithms for the MA problem while guaranteeing same data rates among all users for the RA problem.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a network that consists of $K$ BSs and $I$ mobile terminals (MT). The location of the BSs is modeled as a homogeneous Poisson point process (HPPP) with density $\lambda_{BS}$ and the MTs are scattered based on an independent homogeneous point process with density $\lambda_{MT}$. Each MT is served by its closest BS and the irregularly shaped cell covered by each BS is determined by its Voronoi region.

We assume that the BSs and the MTs are all equipped with a single antenna. In order to avoid intra-cell interference, no subcarrier is assigned to two different MTs within a particular cell. The total bandwidth $B_T$ is divided into $b$ equal frequency subbands composed each of $L$ orthogonal frequency subcarriers.

Due to the non-synchronization among the BSs, inter-cell interference will spread over adjacent subcarriers. Since the timing offset between neighboring BSs is different and is subject to change, the most accurate model for this inter-cell interference is to assume that the timing offset follows a uniform distribution as in [7]. In fact, the interference weight when the modulation technique is either OFDM or FBMC was derived in [7] and can be summarized as

$$V_{OFDM} = [705, 89.4, 22.3, 9.95, 5.6, 3.59, 2.5, 1.84, 1.12] \times 10^{-3}$$
$$V_{FBMC} = [823, 88.1] \times 10^{-3}$$

Let $V = [V_0, \ldots, V_S]$ be the interference weight vector. We then have $S = 8$ and $S = 1$ for the case of OFDM and FBMC, respectively. More concretely, it means that the interference from one subcarrier in the case of OFDM spreads over 17 subcarriers, i.e., 8 subcarriers either side while in the case of FBMC, it spans only over 3 subcarriers.

We further suppose that $L$ is greater than 8 so that inter-cell interference can be from BSs using either the same or adjacent frequency subbands. Let $C$ be the set of all interferences. In the sequel, $u(k,l)$ refers to user $u$ that was allocated the $l$th subcarrier by its serving BS $k$; its instantaneous signal-to-noise-plus-interference (SINR) can be written as

$$\Gamma_{u(k,l)}^l = \frac{P_k^l G_{k,u(k,l)}}{N_0 + \sum_{j \neq k}^K \sum_{l' \in \mathcal{L}} P_{j,l'}^l V_{[l-l']} G_{j,u(k,l)}}$$

(1)
where $P_k^l$ denotes the power that the $k$th BS allocates to the $l$th subcarrier. $G_{k,u(k,l)}^l$ is the channel gain between base station $k$ and user $u$ in subcarrier $l$ and $N_0$, the thermal noise.

The downlink RA optimization problem is formulated as

$$\max_{P_k^l \geq 0, \forall k,l} \min_{a_{l,i,k} \in \{0,1\}} \sum_{l=1}^{L} \sum_{i=1}^{I} a_{l,i,k} r_i^l$$

$$\text{s.t.} \quad \sum_{l=1}^{L} P_k^l \leq P_{\text{max}} \quad \forall k \in \{1, \ldots, K\} \quad (2)$$

where $r_i^l = \log_2(1 + G_i^l)$ is the achievable data rate of the $i$th MT in the $l$th subcarrier and $a_{l,i,k}$ is the subcarrier allocation indicator which is 1 if the $k$th BS allocates the $l$th subcarrier to user $i$ and is zero otherwise. $P_{\text{max}}$ is the total power constraint per BS. It was proved in [4] that the RA problem can be solved recursively via the MA optimization problem. Instead of considering directly the RA, we focus on the MA problem and use the idea developed in [4] to iteratively solve the RA optimization problem formulated in (2).

Given a set of MTs data rates $\{R_1, \ldots, R_I\}$, the downlink MA optimization problem can be written as

$$\min_{P_k^l \geq 0, \forall k,l} \sum_{l=1}^{L} \sum_{i=1}^{I} a_{l,i,k} P_k^l$$

$$\text{s.t.} \quad \sum_{l=1}^{L} a_{l,i,k} r_i^l - R_i \geq 0 \quad \forall i \in \{1, \ldots, I\}$$

$$\sum_{l=1}^{L} P_k^l \leq P_{\text{max}} \quad \forall k \in \{1, \ldots, K\} \quad (3)$$

The problem defined in (3) is a combinatorial optimization problem and therefore of high complexity. A potential suboptimal solution with a tremendous decrease in the complexity is to split the problem into two sub-problems: subcarrier allocation and power allocation. The latter is the main focus of next section.

### III. Convergence Criterion for Power Control

In this section, we derive a sufficient criterion for convergence of iterative algorithms destined to solve power control subproblem pertaining to the optimization problem of (3) provided that the subcarriers’ assignment is properly known. The sufficient condition is summarized in the following Theorem:

**Theorem 1**: Any iterative power control algorithm for the MA problem within multi-user asynchronous networks satisfying

$$\sum_{l \neq k}^{K} \sum_{i \in \mathcal{L}} V_{l|i}(G_{j,u(k,l)}^l) < 1 \quad \forall k \quad (4)$$

converges geometrically to a unique fixed point for every initial power vector. In (4), $\gamma_{u(k,l)}^l$ represents the target SINR for user $u$ that was assigned the $l$th subcarrier by its serving BS.

**Proof**: To start with the proof, let us define the matrices $\mathbf{G}, \mathbf{H}, \mathbf{G}_j \in \mathbb{C}^{L \times K \times L \times K}$ as in (5), shown at the bottom of the page. $G_{ij}$ which is composed of $L \times L$ matrices $O_1$ and $G_{ij}$, is the interference matrix of the entire network. In $G_{ij}$, $O_i$ denotes entry zero matrix and, $G_{ij} = G_{ij}^1 x_{ij} + G_{ij}^2 y_{ij} + G_{ij}^{10} z_{ij}$, where $G_{ij}^{10} \in \mathbb{C}^{L \times L}$ are defined in (5c) and (5d), $x_{ij} = 1_{(B[i] \neq B[j])}$ is the indicator function which is 1 if the $i$th BS and the $j$th BS have been allocated the same frequency subband and zero otherwise. Similarly, $y_{ij} = 1_{(B[j] = B[i]+1)}$ and $z_{ij} = 1_{(B[j] = B[i]-1)}$ are both equal to 1 if the frequency subband allocated to the $j$th BS is adjacent to the one allocated to the $i$th BS. Furthermore, define $N \triangleq N_0 \mathbf{1}_{L \times K \times 1}$ where $\mathbf{1}_{L \times K \times 1}$ is a vector column of entry 1 and, $\mathbf{P}^T \triangleq [P_1^T, \ldots, P_L^T]$, $\mathbf{P}^T$ where $\mathbf{P}^T$ denotes the transpose of the vector $\mathbf{P}$.
Assume that the target SINR is known, the constraint 
\[ \Gamma \geq \gamma_{u(k,l)} \] is equivalent to
\[ P_k G_{k,u(k,l)} \geq \gamma_{u(k,l)} \left( N_0 + \sum_{j \neq k} \sum_{l \in L} V_{l-l'} (G_{j,u(k,l)} P_{j}^l) \right) \]
which can be compactly written as \( \mathbf{HP} \geq \mathbf{\Gamma (N + G_l P)} \), and is equivalent to
\[ \mathbf{P} \geq \mathbf{H^T G_l P + H^T \Gamma N} \] (6)
The power control problem in (6) can be solved by iterative method such as the Gauss–Seidel algorithm [8]. At the \( n \)th iteration, the problem is written as
\[ \mathbf{P}[n] = \mathbf{H^T G_l P}[n-1] + \mathbf{H^T \Gamma N} \] (7)
In the sequel, \( \| \cdot \| \) denotes the infinity norm defined as \( \max_{1 \leq i \leq N} |x_i| \) for a vector \( \mathbf{x} = [x_1, \ldots, x_N] \) and \( \max_{1 \leq i \leq N} \sum_{j=1}^{M} |x_{ij}| \) for a \( N \times M \) matrix \( \mathbf{X} = [x_{ij}]_{i,j=1}^{N,M} \). Given an initial power vector \( \mathbf{P}[0] \), we have
\[ \| \mathbf{P}[n] - \mathbf{P}[n-1] \| = \| \mathbf{H^T G_l P}[n-1] - \mathbf{P}[n-2] \| \leq \| \mathbf{H^T G_l P}[n-1] - \mathbf{P}[n-2] \| \leq \alpha \| \mathbf{P}[n-1] - \mathbf{P}[n-2] \| \leq \alpha^n \| \mathbf{P}[1] - \mathbf{P}[0] \| \] (8)
Hence, for every \( n \geq 0 \) and \( M \geq 0 \), it follows that
\[ \| \mathbf{P}[n + M] - \mathbf{P}[n] \| \leq \sum_{j=1}^{M} \| \mathbf{P}[n + j] - \mathbf{P}[n + j - 1] \| \leq \sum_{j=1}^{M} \alpha^n \| \mathbf{P}[1] - \mathbf{P}[0] \| \leq \alpha^n - \frac{1}{1 - \alpha} \| \mathbf{P}[1] - \mathbf{P}[0] \| \] (9)
where \( \alpha = \max_{1 \leq i \leq L} \gamma_{u(k,l)} \). Such a sequence converges and therefore converges.

To show that the limit point is a unique fixed point, let us rewrite (7) as \( \mathbf{P} = \psi(\mathbf{P}) \), where \( \psi(\mathbf{P}) = \mathbf{H^T G_l P + H^T \Gamma N} \). Next, we show that the mapping \( \psi(\cdot) \) is a contraction mapping. Let \( \mathbf{P}^* \) be a limit point.
\[ \| \psi(\mathbf{P}) - \psi(\mathbf{P}^*) \| = \| \mathbf{H^T G_l (P - P^*)} \| \leq \alpha \| \mathbf{P} - \mathbf{P}^* \| \] Since the iterative method converges and \( \psi(\cdot) \) is a contraction mapping, we then have \( \mathbf{P}^* = \psi(\mathbf{P}^*) \) and conclude that \( \mathbf{P}^* \) is a fixed point and is given by \( \mathbf{P}^* = (\mathbf{I} - \mathbf{H}^\top \mathbf{G_l})^{-1} \mathbf{H}^\top \mathbf{\Gamma N} \) [8]. This concludes the proof.

IV. PROPOSED DECENTRALIZED ALGORITHM

Utilizing the sufficient convergence criterion defined in Theorem 1, we propose a distributed algorithm to efficiently solve (3) provided that subcarriers’ assignment is known by using the method given in [9]. The decentralized algorithm consists of the search of the target SINR on each subcarrier satisfying a per-user target data rate followed by an iterative power control among the BSs.

Let \( \text{IPN}_{k,u(k,l)} = \Delta(N_0 + \sum_{j \neq k} \sum_{l \in L} V_{l-l'} (G_{j,u(k,l)} P_{j}^l)) \) denote the normalized interference plus noise per user on each assigned subcarrier. It can be measured at the MTs and fed back to their serving BS. Since the inequality constraint \( \Gamma \geq \gamma_{u(k,l)} \) will be tight when optimality is reached, it can therefore be written as \( P_{k}^l = \gamma_{k,u(k,l)} \text{IPN}_{k,u(k,l)} \).

The target SINR search algorithm is executed in parallel at each BS by solving the following optimization problem:
\[ \min_{l=1}^{L} \gamma_{k,u(k,l)} \text{IPN}_{k,u(k,l)} \]
\[ \text{s.t.} \quad \sum_{n=1}^{K} \log_2 (1 + \gamma_{n}^l) - R \geq 0 \quad \forall i \in \{1, \ldots, \text{MT}[k]\} \]
\[ \gamma_{k,u(k,l)} \leq \frac{G_{k,u(k,l)}}{\sum_{j \neq k} \sum_{l' \in L} V_{l-l'} (G_{j,u(k,l)} P_{j}^{l'})} - \epsilon \quad \forall l \] (10)
where for each user \( i \), \( L_{\text{sub}[i]} \) is the number of subcarriers allocated by its serving BS and \( \text{MT}[k] \) denotes the total number of mobile stations served by the \( k \)-th BS and \( \epsilon \) is an infinitesimal positive constant used in order to meet the convergence criterion. Problem (10) is a convex optimization problem and can therefore be solved using CVX [10]. Once the target SINR per subcarrier is known, the power control problem in (7) is solved iteratively in parallel at the BSs.

In order to solve the RA optimization problem in (2), we recursively solve the MA problem described in (3). More specifically, we first solve (10) followed by the power control in (7) then verify after convergence of the power control algorithm if the total power per BS is less than \( P_{\text{max}} \). If so, the per-user prescribed rate requirement \( R \) is updated by adding \( \Delta \) to its current value. Once one of the BS violates the power constraint, the algorithm halts while achieving maximum fairness of one among all users since they get the same rate. Our proposed algorithm for problem (2) is summarized in Algorithm 1.

**Algorithm 1 Proposed Algorithm**

1. **Input** The solution of subcarrier allocation subproblem using [9];
2. Initialize the data rate \( R \) and set \( n = 0 \);
3. repeat
4. \( n := n + 1 \);
5. \( R[n] := R[n-1] + \Delta \);
6. Initialize \( P_k^l \forall k, l \) and set \( t = 0 \);
7. repeat
8. \( t := t + 1 \);
9. Find \( \{\gamma_{k,u(k,l)}\} \) by solving (10);
10. Set \( iteration := 0 \);
11. repeat
12. \( iteration := iteration + 1 \);
13. Solve (7);
14. until convergence;
15. Update \( P_k^l \forall k, l \) by using \( P_k^l = \gamma_{k,u(k,l)} \text{IPN}_{k,u(k,l)} \);
16. until \( t = T \);
17. until \( \exists k \in \{1, \ldots, K\} \| \sum_{n=1}^{L} P_k^l > P_{\text{max}} \);
18. Output \( R = R[n-1] \).
V. NUMERICAL RESULTS

The following numerical results are conducted using Monte Carlo simulation by averaging over 500 realizations. All channel realizations between the BSs and MTs are assumed to be i.i.d. static Rayleigh fading with the path loss model ITU-R P1411-7 [11] and a carrier frequency of 700 MHz. The shadowing’s standard deviation is 9 dB and the total area of communication is given to be 20 km$^2$. Unless otherwise stated, the BS density is $\lambda_{BS} = 1$ meaning that there is on average 20 BSs in the network.

The total bandwidth $B_T$ is chosen to be 1.4 MHz. It is divided into 6 frequency subbands, i.e., $b = 6$ which are randomly allocated to the BSs. Each BS is communicating over $L = 12$ subcarriers having each a bandwidth of 15 kHz. The total power constraint per BS is given to be 43 dBm. $\Delta$ is equal to 4 kbits/s and $\epsilon$ is chosen to be $10^{-8}$. The thermal noise has a spectral density of $-174$ dBm/Hz.

In order to find a benchmark for our result, we compare with the perfect synchronization case denoted as PS. The performance and convergence properties of Algorithm 1 are examined in Fig. 1 under different modulation techniques. Fig. 1(a) shows the convergence behavior, in terms of the network total transmit power versus number of iterations, of the iterative power control algorithm for a fixed user rate $R = 150$ kbits/s.

Fig. 1(b) demonstrates the robustness of the proposed convergence criterion. The iterative power control algorithm converges to the same limit point regardless of the value of the initial power ($P_{\text{max}}/L$, 0 and random) for a particular modulation scheme.

In Fig. 1(c), a fair comparison between the proposed Algorithm 1 and the waterfilling algorithm is provided in terms of maximum minimum data user rates versus the MT density. We see clearly that our proposed algorithm always achieves better rate performance than the waterfilling algorithm regardless of the modulation scheme considered. There is a gain varying from 40.12% to 46.31% between the performance of the proposed algorithm and the waterfilling algorithm in the case of FBMC and 35.17% to 41.7% for OFDM.

VI. CONCLUSION

In this letter, we proposed a fully distributed and general convergence criterion for iterative power control algorithm for MA problem within asynchronous networks with multi-carrier modulation techniques. The efficiency of the proposed convergence criterion was validated through numerical experiments. It was demonstrated via numerical results that algorithms solving the RA problem that utilize our proposed convergence condition over the iterative waterfilling achieve better performance in terms of higher maximum minimum rate. Furthermore, the simulation results highlighted the advantages of using FBMC instead of OFDM as a modulation scheme.

REFERENCES