Abstract—This paper addresses resource allocation for weighted sum throughput maximization (WSTM) in multicell orthogonal frequency-division multiple-access (OFDMA) networks. Two suboptimal algorithms with polynomial-time complexity are provided. First, we determine a graph-based subcarrier allocation algorithm allowing joint transmission of two interfering links whenever such a transmission fulfills the pairwise WSTM objective. An analytical expression of the optimal power allocation with two interfering cells in single-carrier transmission is obtained, and the capacity region study serves as the basis to build the interference graph. Second, we present a distributed power-control algorithm suitable for any signal-to-interference-plus-noise ratio (SINR) regime. It rejects users and subcarriers with weighted SINR that is too low before operating in the high SINR regime for the remaining users and subcarriers. Both algorithms and their combination are assessed via dynamic simulations, where the weight of each user is proportional to its queue length. They are shown to significantly decrease resource consumption and efficiently balance users’ queue lengths.

Index Terms—Capacity region, intercell interference, orthogonal frequency-division multiple access (OFDMA), power allocation, subcarrier allocation.

I. INTRODUCTION

Orthogonal frequency-division multiple access (OFDMA) offers a fine granularity for resource allocation via subcarrier and power allocation. Orthogonal subcarrier access within each cell removes intracell interference. However, in multicell networks, adjacent cells may be using the same subcarriers for unrelated transmissions. Thus, each subcarrier can be viewed as a potential interference channel. Network-wide resource allocation in OFDMA should consequently consider intercell interference on top of intracell constraints, such as downlink sum power limitation and adjacent subcarrier allocation.

In this paper, the studied resource-allocation problem is weighted sum throughput maximization (WSTM) in a downlink multicell OFDMA network. It is based on cross-layer results for the flat-fading Gaussian multiaccess channel (MAC) [1], where the resource-allocation policy is determined with the objective of bounding the queue lengths of all involved users. When the queue arrival rates are unknown, such a policy is called throughput optimal. The authors in [1] show that throughput optimal resource allocation is obtained by solving the WSTM problem with the method from [2], where the weight of each user is proportional to its queue length. Successive decoding is performed over users sharing the channel, from the shortest queue to the longest queue. A similar result is derived for the broadcast channel by applying the solution from [3] and [4]. The information-theoretic results obtained on the MAC and broadcast channels may not extend to the interference channel, where rate-achieving strategies such as successive decoding cannot be used. Nevertheless, the WSTM resource-allocation strategy seems intuitively well suited to efficiently balance the queues of users in any multiuser scenario. Therefore, we study the WSTM problem and consider queue length management as an example of application.

A thorough description of the state of the art on WSTM is provided in the next section. Although this NP-hard combinatorial problem has extensively been investigated in the literature, only some subcases have been fully addressed. Even in the simplest case with two interfering users, the WSTM problem has not been solved in terms of analytical throughput expressions. In this paper, we study this specific subcase and then provide low-complexity subcarrier allocation and power-control algorithms for WSTM in a downlink multicell OFDMA. Our contributions are as follows:

1) First, the capacity region of two interfering users in single-carrier transmission is fully characterized. The optimal resource allocation for WSTM is analytically derived, and a binary suboptimal criterion is deduced. This criterion is used to build an interference graph for OFDMA networks. A graph-based subcarrier-allocation algorithm, aiming at maximizing the pairwise weighted sum throughput, is then obtained.

2) Second, a distributed power-control algorithm is determined for the WSTM problem. Its main advantage with respect to the current state of the art is that it is suitable for any signal-to-interference-plus-noise ratio (SINR) regime, since it first identifies which users and subcarriers should be inactive because of their weighted SINR that is too low, and then operates in the high SINR regime for the remaining users and subcarriers.

3) Finally, the proposed algorithms, which are used jointly or separately, are assessed via dynamic simulations, where the weight of each user is proportional to its queue length. The objective is to evaluate if, in a scenario...
with unequal user traffic load, the algorithms can prevent user queues from increasing without bounds. Consequently, the performance-evaluation concerns not only the achieved weighted sum throughput but the final queue lengths as well. It is shown that our algorithms are both very efficient in terms of WSTM and queue length fairness.

This paper is organized as follows: Section II describes the system model and the state of the art. Subcarrier allocation is studied in Section III, and power control is studied in Section IV. Section V provides assessments of the algorithms via static and dynamic simulations. Finally, Section VI concludes this paper.

II. SYSTEM MODEL AND STATE OF THE ART

A. System Model

We consider the downlink transmission in a network $\mathcal{N}$ consisting of $K_N$ users and $N_{BS}$ base stations (BSs) using OFDMA. Each BS has $L_{SC}$ subcarriers available for data transmission. The bandwidth per subcarrier is $B_{SC}$. Let $BS[k]$ be the index of the BS serving user $k$. The data rate of user $k$ is

\[ R_k = \sum_{l=0}^{L_{SC}} B_{SC} \log_2 \left(1 + \frac{G'_{BS[k],k} P'_l}{N_0 + \sum_{n_{BS} \neq BS[k]} G'_{n_{BS},k} P'_{n_{BS}}} \right) \]

where $\Theta_k$ is the set of subcarriers assigned to user $k$ by BS $[k]$, $P'_l$ is the power transmitted to user $k$ by BS $[k]$ in subcarrier $l$, and $G'_{n_{BS},k}$ is the channel coefficient between BS $n_{BS}$ and user $k$ in subcarrier $l$ (including propagation loss, shadowing, and Rayleigh fading). $N_0$ is the variance of the additive white Gaussian noise. To simplify notations, we equivalently denote $P'_l$ as $P'_{BS[k]}$ if subcarrier $l$ is allocated to user $k$ by its serving BS.

Let $w_k$ be the weight of user $k$. Throughout this paper, we consider normalized weights, i.e., $\sum_{k=1}^{K_N} w_k = 1$. The WSTM problem is

\[ \max_{\{P, \Theta\}} \sum_{k=1}^{K_N} w_k R_k \]

subject to

\[ \sum_{l=1}^{L_{SC}} P'_{n_{BS}} \leq P_{max} \quad \forall n_{BS} \in \{1, \ldots, N_{BS}\} (C_1) \]

\[ P'_{n_{BS}} \geq 0 \quad \forall (n_{BS}, l) \in \{1, \ldots, N_{BS}\} \times \{1, \ldots, L_{SC}\} \quad (C_2) \]

\[ \Theta_k \cap \Theta_j = \emptyset \quad \forall (k,j), \quad BS[k] = BS[j], \quad k \neq j \quad (C_3) \]

where ($C_1$) is the sum power constraint per BS, with $P_{max}$ defined as the maximum BS transmit power, and ($C_3$) is the OFDMA subcarrier allocation orthogonality constraint. The optimization variables are the set of power values $P$ per BS and per subcarrier and the set of subcarriers $\Theta$ per BS.

B. State of the Art

Problem (2) is an NP-hard combinatorial optimization problem. Several subcases of (2) have been solved in the literature. In single-carrier transmission, (2) only amounts to a power-allocation problem. As $S(P) = \sum_{k=1}^{K_N} w_k R_k$ is positive, continuous, and differentiable, the optimization problem has a global maximum in $[0, P_{max}]$. The optimal power allocation for the sum throughput maximization problem has analytically been derived in [5]–[7] when $N_{BS} = 2$. This result was obtained by studying the capacity region. However, these papers did not investigate the impact of the weights on power allocation. Since the single-carrier problem [see (2)] is not convex, because $S(P)$ is not a concave function [8], the distributed iterative algorithm proposed in [9] may only lead to local optima. In the high SINR regime, when $\log_2(1 + \text{SINR}) \approx \log_2(1 + \text{SINR})$, the single-carrier WSTM problem belongs to the class of geometric programming [10]. Consequently, it is equivalent to a convex optimization problem and has a unique global optimum that can be obtained in a distributed way [11]–[13].

In the high SINR regime, WSTM is also solved for single-user orthogonal frequency-division multiplexing through a decomposition in dual space [12]. In OFDMA, the discrete subcarrier assignment constraint renders the problem far more complex [14], [15]. Joint allocation of subcarriers and power has been studied in [16] for discrete multitone systems. Their conclusions directly apply to uplink multicell OFDMA and have recently been extended to downlink multicell OFDMA in [17]. They show that the duality gap of the WSTM problem [see (2)] tends to zero when the number of subcarriers goes to infinity. Consequently, under this assumption, the joint resource allocation problem can be solved via Lagrange dual decomposition. In single-cell downlink OFDMA, this property is even verified with a finite number of subcarriers [18]. However, this conclusion no longer stands in multicell downlink OFDMA. The Lagrange dual decomposition method may then be suboptimal, although it is very complex and not distributed [17]. In this paper, we consider resource-allocation algorithms with low complexity, and we therefore assume that subcarrier allocation and power allocation are performed in sequence.

Many suboptimal algorithms have been proposed in the literature for multicell OFDMA WSTM [19]–[26]. In [25] and [26], an iterative water-filling method [27] for sum throughput maximization is determined, where users are prevented from transmitting as soon as their SINR falls below a threshold. A similar approach is used in our power-control algorithm, but low SINR users are then simultaneously rejected at the end of a first phase. Finally, the influence of higher layers on power allocation has been investigated in [13] for multihop networks. In this paper, we evaluate the impact of dynamic weights on WSTM and assess the performances of the proposed algorithms not only in terms of achieved weighted sum throughput but in terms of fairness between users as well.

III. WEIGHTED SUM THROUGHPUT MAXIMIZATION SUBCARRIER ALLOCATION

This section is concerned with subcarrier allocation for the WSTM problem. It first determines the optimal solution of
WSTM in the two-cell single-carrier case. A binary suboptimal criterion is then derived. When combined with power control, this criterion leads to a solution very close to the optimum, as shown in Section V-A. It is therefore used in multicell OFDMA to build an interference graph and obtain a polynomial complexity algorithm.

A. Analytical Solution for Single-Carrier WSTM With Two Cells

We investigate here the single-carrier case with \( N_{\text{BS}} = 2 \). To simplify notations, we remove the subcarrier index \( k \), consider normalized rates \( (R_{\text{SC}} = 1) \), and denote the channel gain between user \( i \) and its serving BS as \( G_{\text{BS}[i,k]} = G_{k,k} \) \( \forall k \in \{1,2\} \).

1) Optimal Solution: The data rate of user 1 interfered by user 2 is

\[
R_1 = \log_2 \left( 1 + \frac{G_{1,1}P_1}{N_0 + G_{2,1}P_2} \right),
\]

with \( 0 \leq P_1 \leq P_{\text{max}} \), and \( 0 \leq P_2 \leq P_{\text{max}} \).

The capacity region is defined as the set of simultaneously achievable data rates \( (R_1, R_2) \), when intercell interference is treated as noise, and when each cell has a maximum power constraint.

\( R_1 \) can be expressed as a function of \( P_1 \) and of the interfering link’s rate \( R_2 \) as

\[
R_1 = \tilde{f}_1(R_2, P_1) = \log_2 \left( 1 + \frac{G_{1,1}P_1}{N_0 + G_{2,1}P_2} \right).
\]

Let us set \( a = N_0 + (G_{2,1}/G_{2,2})(2R_2 - 1)N_0 \) and \( b = (G_{2,1}/G_{2,2})(2R_2 - 1)G_{1,2} \). Then, for any value of \( R_2 \), we have

\[
\frac{\partial \tilde{f}_1(R_2, P_1)}{\partial P_1} = \frac{aG_{1,1}}{\log_2(2)(a + bP_1)(a + bP_1 + G_{1,1}P_1)} > 0.
\]

Consequently, the maximum of \( \tilde{f}_1(R_2, P_1) \) is obtained when \( P_1 = P_{\text{max}} \), and the capacity region is determined by the following set of inequalities:

\[
R_1 \leq \tilde{f}_1(R_2) \text{ and } R_2 \leq f_2(R_1)
\]

where \( f_1(R_2) = \tilde{f}_1(R_2, P_{\text{max}}) \), and \( f_2(R_1) = \tilde{f}_2(R_1, P_{\text{max}}) \). Two examples of capacity regions are represented in Figs. 1 and 2, with \( N_0 = -105 \) dBm, \( P_{\text{max}} = 43 \) dBm, and \( G_{1,1} = G_{2,2} = 10^{-12} \).

In the first example, the interfering channel gains are \( G_{1,2} = G_{2,1} = 10^{-13} \), leading to a concave capacity region. In the second example, \( G_{1,2} = G_{2,1} = 10^{-14} \), and the capacity region is convex.

The weighted sum throughput is \( S(P) = w_1R_1 + w_2R_2 \), with \( w_1 + w_2 = 1 \). Let us assume that \( w_1 \geq w_2 \). If the solution that maximizes \( S(P) \) is different from the boundaries \( P_b = \{(P_{\text{max}}, 0), (0, P_{\text{max}}), (P_{\text{max}}, P_{\text{max}})\} \), then the corresponding line \( R_1 = (S(P)/w_1) - (w_2/w_1)R_2 \) is tangent to \( f_1 \). Consequently, it is obtained by solving equation

\[
f_1'(R_2) = -(w_2/w_1) \). There are two cases regarding the \( f_1 \) curvature: Either \( f_1 \) is convex on \([0, +\infty]\), or \( f_1 \) has only one inflexion point \( R_{\text{inflex}} \). In the second case, \( f_1 \) is first concave between \([0, R_{\text{inflex}}] \) and then convex on \([R_{\text{inflex}}, +\infty]\).

If \( f_1 \) or \( f_2 \) is convex on \([0, +\infty]\), then the capacity region is concave, and the solution of WSTM is \((P_{\text{max}}, 0) \) or \((0, P_{\text{max}}) \). If \( f_1 \) and \( f_2 \) are concave on \([0, R_{\text{inflex}}] \) and \([0, R_{\text{inflex}}] \) respectively, then the capacity region is convex on \([0, R_{\text{inflex}}] \times [0, R_{\text{inflex}}] \). We now consider this case.

The candidate solutions of \( f_1'(R_2) = -(w_2/w_1) \) are \( R_{2,a} = \log_2(v - \sqrt{\alpha}/\chi) \) and \( R_{2,b} = \log_2(v + \sqrt{\alpha}/\chi) \), where

\[
v = (w_1 - w_2)G_{1,1}G_{2,2}P_{\text{max}} - 2w_2G_{2,2}N_0 + 2w_2G_{2,1}(N_0 + G_{1,2}P_{\text{max}}) + 4w_1w_2G_{1,2}G_{2,1}P_{\text{max}} + 4w_1w_2N_0(G_{2,1} - G_{2,2}) \]
\]

Only \( R_{2,a} \) may be suitable, because \( R_{2} \) must be in the concave area of \( f_1 \). The corresponding power value is \( P^*_2 = ((N_0 + G_{1,2}P_{\text{max}}))/G_{2,2})(1 - (v - \sqrt{\alpha}/\chi)) \). This solution is valid if \( 0 \leq P^*_2 \leq P_{\text{max}} \). Let \( [x]^{+} = \max(0, x) \).
The slope of \( f_1 \) in its concave area belongs to 
\[ \left[ f_1'(R_2^{\text{inflex}}), f_1'(0) \right] \in [-1, 0]. \]
Let us define \( M_1 = -f_1'(R_2^{\text{inflex}}) \) and 
\[ m_1 = -f_1'(0) = \frac{G_{2,1}G_{1,1}P_{\text{max}}(N_0 + G_{1,2}P_{\text{max}})}{G_{2,2}N_0(N_0 + G_{1,1}P_{\text{max}})}. \]

There is a solution to \( f_1'(R_2^*) = -(w_2/w_1) \) in the concave area of \( f_1 \) if \( (w_2/w_1) \in [m_1, M_1]. \) In this case, the solution of WSTM is \((P_{\text{max}}, P_2^*)\), where 
\[ P_2^* = \min \left( \left[ \left( \frac{N_0 + G_{1,2}P_{\text{max}}}{G_{2,2}} \right) \left( 1 - \frac{w - \sqrt{\alpha}}{\chi} \right) \right]^{+} ; P_{\text{max}} \right). \]

If \((w_2/w_1) \notin [m_1, M_1], \) then the solution is in \( \mathcal{P}_b. \) To conclude, when \( w_1 \geq w_2, \) the optimal solution of WSTM belongs to the following set: 
\[ \mathcal{P}_{\text{joint}} = \{(P_{\text{max}}, 0), (0, P_{\text{max}}), (P_{\text{max}}, P_{\text{max}}), (P_{\text{max}}, P_2^*)\}. \]

2) Binary Suboptimal Solution: The capacity region study and the optimal solution derivation for systems with more than two cells become rapidly intractable. We propose to circumvent this limitation by considering the pair-wise weighted sum throughput when \( N_{\text{BS}} \geq 2 \) instead of the total weighted sum throughput. For that purpose, we need to determine a binary criterion for resource allocation when \( N_{\text{BS}} = 2. \) It should indicate if the weighted sum throughput is higher when both users are transmitting or when one user is inactive. This criterion will then enable us to build an interference graph, which will be valid whatever the number of interfering cells. To derive this criterion, we study the capacity region when \( N_{\text{BS}} = 2. \) It is convex on \([0, R_2^{\text{inflex}}] \times [0, R_1^{\text{inflex}}]\) if \( f_1'(0) \leq 0 \) and \( f_2'(0) \leq 0. \)

\[ f_1''(0) = -\log_e(2)G_{2,1}G_{1,1}P_{\text{max}}(N_0 + G_{1,2}P_{\text{max}})\psi \]
\[ (G_{2,2})^2(N_0)^2(N_0 + G_{1,1}P_{\text{max}})^2 \]

where 
\[ \psi = G_{2,2}N_0(N_0 + G_{1,1}P_{\text{max}}) \]
\[ - G_{2,1}(N_0 + G_{1,2}P_{\text{max}})(2N_0 + G_{1,1}P_{\text{max}}). \]

Consequently, \( f_1''(0) \leq 0 \) is equivalent to \( A_{1,2} \geq 0, \) where 
\[ A_{1,2} = \frac{G_{2,2}N_0}{N_0 + G_{1,2}P_{\text{max}}} - G_{2,1}(2N_0 + G_{1,1}P_{\text{max}}). \]

Let us consider the case when \( P_1 = P_{\text{max}} \) is optimal. If the capacity region is convex on \([0, R_2^{\text{inflex}}] \times [0, R_1^{\text{inflex}}], \) then any tangent to \( f_1 \) at a point in \([0, R_2^{\text{inflex}}]\) will cross the axis at a higher point \( S(P_1)/w_1 \) than \( f_1(0) = \log_2(1 + (G_{1,1}P_{\text{max}}/N_0)). \) In this case, and if a solution to \( f_1'(R_2^*) = -(w_2/w_1) \) exists in \([0, R_1^{\text{inflex}}], \) the weighted sum throughput at \( R_2^* \) is higher than the weighted sum throughput obtained when the transmission is limited to user 1.

The weighted sum throughput can thus be increased by assigning subcarriers to avoid having \( f_1''(0) > 0 \) and \( f_2''(0) > 0, \) or equivalently, \( A_{1,2} < 0 \) and \( A_{2,1} < 0. \) It should be noted that criterion (8) does not depend on the weight values, as it only characterizes the convexity of the capacity region, which is independent of the weights. However, we have seen that if \((w_2/w_1) \in [0, m_1] \) or \((w_1/w_2) \in [0, m_2], \) then the optimal solution is in \( \mathcal{P}_b. \) Since in this case either \( w_1 \) or \( w_2 \) is very low, joint transmission is most certainly not the optimal solution. We thus add a second condition, which is dependent on the weights, to evaluate if the joint transmission of users 1 and 2 leads to WSTM. To conclude, the binary criterion for optimality of joint transmission is 
\[ A_{1,2} \geq 0, \quad A_{2,1} \geq 0, \quad \frac{w_2}{w_1} \geq m_1, \quad \text{and} \quad \frac{w_1}{w_2} \geq m_2. \]

B. Graph-Based Subcarrier Allocation for OFDMA

We now extend our results to subcarrier allocation for OFDMA with \( N_{\text{BS}} > 2 \) and \( L_{\text{SC}} > 1 \) subcarriers. For each couple of links, the weighted sum throughput is higher with joint transmission than with separate transmission, provided that the conditions in (9) are fulfilled. The proposed subcarrier-allocation algorithm assigns, on the same subcarrier, only the users that verify these conditions per pair. It uses an interference graph derived from the conditions in (9). Let us define the following for subcarrier \( l \in \{1, \ldots, L_{\text{SC}}\} \) and users \((k,j)\) in \( \{1, \ldots, K_{\text{SC}}\} \times \{1, \ldots, K_{\text{SC}}\}, \)
\[ A_{l,k,j} = \frac{G_{l,1,j}G_{j,1}N_0}{N_0 + G_{l,j,k}P_{\text{max}}} - G_{l,1,j}G_{j,1}P_{\text{max}}. \]

The interference matrix \( F_l \) representing the interference graph on subcarrier \( l \) is built as follows: \((F_l)_{(k,j)} = 1 \) if \( A_{l,k,j} < 0, \) \((k,j) < m_1, \) or \((w_k/w_j) < m_2 \) (pairwise interference condition), or if \( k \) and \( j \) are different users served by the same BS (OFDMA intracell orthogonality condition). Else, \((F_l)_{(k,j)} = 0. \) If vertex \( k \) and \( j \) are adjacent in the interference graph (i.e., \((F_l)_{(k,j)} = 1, \)) then their simultaneous transmission in the same subcarrier is forbidden, either because this would decrease their weighted sum throughput or because they are orthogonal users belonging to the same cell.

The users that are allowed simultaneous transmission in the same subcarrier are obtained through graph coloring. To limit the complexity, we consider the interference graph \( F \) on the average channel (including propagation loss and shadowing), and therefore, graph coloring is independent of the subcarriers. Subcarrier allocation is detailed in Algorithm 1. On each subcarrier \( l, \) the algorithm determines the user \( k^* \) with the highest priority and then allocates \( l \) to the users of the interfering cells that have the same color as \( k^*. \) The priority of user \( k \) is based on its weighted rate considering the SNR \( w_k \log_2(1 + \text{SNR}_k^l) \)
with SNR\textsubscript{k} = G\textsubscript{BS[k],k} P\textsubscript{max}/N\textsubscript{0} L\textsubscript{SC}. \Delta_k is defined as the set of users in \{1, \ldots, K\} that have the same color as k.\footnote{It should be noted that we have tested, via simulations, objectives other than the maximization of \(w\) log\((1 + \text{SNR}_{k})\) (maximization of the SNR and of the weight, separately) and that this objective leads to the highest weighted sum throughput.}

**Algorithm 1** Graph-based subcarrier allocation

_Initialization:_ Build the interference graph F and perform graph coloring.

_for l = 1 to \(L_{SC}\) do

Allocate subcarrier \(l\) to user \(k^* = \arg\max_{k \in \{1, \ldots, K\}} w_k \log_2(1 + \text{SNR}_k)\)

_for BS = 1 to \(N_{BS}\), BS \(\neq BS[k^*]\) do

If \(\Delta_{k^*} \neq \emptyset\), allocate subcarrier \(l\) to user \(j^* = \arg\max_{j \in \Delta_{k^*}} w_j \log_2(1 + \text{SNR}_j)\)

end for

end for


When greedy heuristic Degree SATU Rat ion (DSATUR) [28] is used for graph coloring, the complexity of graph-based subcarrier allocation is polynomial time in \(O(N_{BS} K^3)\), where \(K\) is the number of users per cell. Distributed graph-coloring algorithms could be used instead of DSATUR for implementation in distributed networks. A review of the complexity of these algorithms is provided in [29].

**IV. WEIGHTED SUM THROUGHPUT MAXIMIZATION DISTRIBUTED POWER CONTROL**

This section describes a distributed power control algorithm in two phases for solving the WSTM problem [see (2)] when subcarrier allocation is set. It consists of identifying the links with weighted SINR that is too low, rejecting them, and then operating in high SINR regime for the remaining ones.

A. Phase I

In Phase I, the WSTM problem is solved on each subcarrier independently. The optimization problem on subcarrier \(l \in \{1, \ldots, L_{SC}\}\) is

\[
\max_{P^l} \sum_{k \in \Omega_l} \alpha_k \log_2 \left(1 + G_{BS[k],k} P^l_k / I^l_k \right)
\]

subject to \(P^l_k \in [P_{\min}, P_{\max}]\) \(\forall k \in \Omega_l\) (11)

where the optimization variable is \(P^l\), which is the vector of transmitted powers in subcarrier \(l\). \(\Omega_l\) is the set of interfering users, \(I^l_k\) is the interference plus noise received by user \(k\), and \(\gamma^l_k = G_{BS[k],k} P^l_k / I^l_k\) is the SINR of user \(k\) in subcarrier \(l\). \(P^l_k\) is allowed to vary between a minimum power \(P_{\min} \geq 0\) and the maximum power per cell \(P_{\max}\). To account for the number of allocated subcarriers, we consider the weight per user and subcarrier \(\alpha_k = w_k / \Theta_k\).

As explained in Section II-B, problem (11) is not convex but has a global maximum. The local maxima of the function \(\Gamma^l\):

\[
P^l \mapsto \sum_{k \in \Omega_l} \alpha_k \log_2(1 + (G_{BS[k],k} P^l_k / I^l_k)) \]

consist, at each direction \(k\), of either the boundaries \{\(P_{\min}, P_{\max}\)\} or of the unique interior point that fulfills \(\partial \Gamma^l (P^l) / P^l = 0\) given by

\[
P^l_k = \frac{\alpha_k \gamma^l_k}{(1 + \gamma^l_k) \left(\sum_{j \in \Omega_l, j \neq k} G_{BS[j],j} \alpha_j \gamma^l_j \right)}.
\] (12)

A local optimum of (11) is obtained by using an iterative process on \(P^l\), where at each iteration \(T + 1\), \(P^l_k(T + 1)\) is either set to one of the boundaries or to (12), according to the following rule:

\[
P^l_k(T + 1) = P_{\min}, \quad \text{if } X^l_k(P^l(T)) \leq P_{\min}
\]

\[
P^l_k(T + 1) = P_{\max}, \quad \text{if } X^l_k(P^l(T)) \geq P_{\max}
\]

\[
P^l_k(T + 1) = X^l_k(P^l(T)), \quad \text{otherwise}
\] (13)

where

\[
X^l_k(P^l(T)) = \frac{\alpha_k \gamma^l_k (P^l(T))}{1 + \gamma^l_k (P^l(T)) \left(\sum_{j \in \Omega_l, j \neq k} G_{BS[j],j} \alpha_j \gamma^l_j (P^l(T))\right)}.
\] (14)

and \(\gamma^l_j (P^l(T))\) is the interference information of user \(j\) in subcarrier \(l\), e.g.,

\[
\gamma^l_j (P^l(T)) = \frac{\alpha_j \gamma^l_j (P^l(T))}{1 + \gamma^l_j (P^l(T))}.
\] (15)

\(\gamma^l_j (P^l(T))\) is sent to all interfering BSs so that they can take into consideration the interference that they generate to user \(j\) when updating their power level. Indeed, in (14), \(G_{BS[k],j} \gamma^l_j (P^l(T))\) may be interpreted as the price charged by user \(j\) to BS\(k\) for the interference that it receives. The power-control algorithm thus tends to balance the interference levels among users.

The iterative process, which is detailed in Phase I of Algorithm 2, converges due to the continuity and differentiability of the function \(\Gamma^l\). The proof is omitted here for brevity, but it is similar to the proof detailed in [9]. The final state may be a local optimum. The simulation results show that, when starting from \(P_{\max}\) for all users, if \(P_{\min} = 0\), then users whose weighted SINR tends to 0 reach this value in a few iterations. This set of initial values thus seems to be adapted to Phase I, whose objective is to determine users with low weighted SINR.

B. High Weighted SINR Condition

At the end of Phase I, a test is used on each subcarrier \(l \in \{1, \ldots, L_{SC}\}\) to check if users have a weighted SINR that is high enough. For user \(k \in \Omega_l\), the high weighted SINR condition with precision \(\beta\) is verified if and only if \(\|\alpha_k \log_2(1 + \gamma^l_k) - \alpha_k \log_2(\gamma^l_k)\| \leq \beta\). This is equivalent to

\[
\gamma^l_k \geq \frac{1}{2} \left(\frac{\beta}{\alpha_k} - 1\right).
\] (16)

If this condition is not fulfilled, then user \(k\) is no longer considered active on subcarrier \(l\) and is removed from \(\Omega_l\).

Authorized licensed use limited to: Emmanuelle Vivier. Downloaded on February 23,2010 at 03:36:51 EST from IEEE Xplore. Restrictions apply.
C. Phase II

This phase is an adaptation of the dual asynchronous distributed pricing (DADP) algorithm for the multichannel case from Huang et al. [12], where we introduce an additional constraint on the minimum power to ensure that the high weighted SINR condition is always verified. The optimization problem in the high SINR regime is:

\[
\max_p \sum_{l=1}^{L_{SC}} \sum_{k \in \Omega_l} \alpha_k \log_2 \left( \frac{C_{BS[k],k}^l p_k^l}{I_k} \right)
\]

subject to
\[
L_{SC} \sum_{l=1} P_{n_{BS}}^l \leq P_{\text{max}} \quad \forall n_{BS} \in \{1, \ldots, N_{BS}\}(C_1)
\]
\[
P_k^l \geq P_{k,\text{min}}^l \quad \forall l \in \{1, \ldots, L_{SC}\} \quad \forall k \in \Omega_l(C_2)
\]

where \(P_{k,\text{min}}^l > 0\) is the minimum allowed power for user \(k\) in subcarrier \(l\).

When the minimum power per subcarrier is fixed to a strictly positive value, problem (17) belongs to the class of geometric programming [10] and has a unique global optimum that can be obtained by an iterative process. Introducing the adaptive constraint \(P_k^l \geq P_{k,\text{min}}^l\) into the optimization problem [see (17)] may bring the convergence of this iterative process into question. However, simulation results show that when the high weighted SINR precision \(\beta\) is accurately set, \(P_{k,\text{min}}^l\) rapidly converges to a fixed value that is defined as

\[
P_{k,\text{min}}^l = \frac{1}{2 \pi r - 1} \frac{I_k}{G_{BS[k],k}^l}.
\]

As the convergence of the iterative process is independent of the initial values, we will hereunder consider the stable state when \(P_{k,\text{min}}^l\) has reached its fixed value. The convergence proof from [12] thus remains valid.

Problem (17) is solved by Lagrangian relaxation. The sum power constraint is relaxed by introducing a dual price per BS, e.g., \(\mu_{BS} \forall n_{BS} \in \{1, \ldots, N_{BS}\}\). In the dual space, the initial problem is separated into \(L_{SC}\) problems, i.e., one per subcarrier. For \(l \in \{1, \ldots, L_{SC}\}\), we have

\[
\max_{p^l} \sum_{k \in \Omega_l} \alpha_k \log_2 \left( \frac{C_{BS[k],k}^l p_k^l}{I_k} \right) - \mu_{BS[k]} P_k^l
\]

subject to \(P_k^l \in [P_{k,\text{min}}^l, P_{\text{max}}] \quad \forall k \in \Omega_l\). (19)

Each problem is locally convex and has a global optimum. The maximum of the function \(\Phi^l : p^l \mapsto \sum_{k \in \Omega_l} \alpha_k \log_2(G_{BS[k],k}^l p_k^l/I_k) - \mu_{BS[k]} P_k^l)\) consists, at each direction \(k\), of either the boundaries \(\{P_{k,\text{min}}^l, P_{\text{max}}\}\) or of the only interior point that fulfills \(\partial \Phi^l(p^l)/\partial p_k^l = 0\) given by

\[
P_k^l = \frac{\alpha_k}{\sum_{j \in \Omega_l, j \neq k} G_{BS[j],j}^l \frac{\alpha_j}{I_j} + \mu_{BS[j]}}
\]

where we have introduced the normalization factor \(\text{log}_e(2)\) inside the dual price of the BS serving user \(k\), \(\mu_{BS[k]}\). We use an iterative process similar to [9]. At each iteration \(T + 1\), \(P_k^l(T + 1)\) is either set to one of the boundaries or to (20), according to the following rule:

\[
\begin{align*}
P_k^l(T + 1) &= P_{k,\text{min}}^l, \quad \text{if } Y_k^l \left( P_k^l(T) \right) \leq P_{k,\text{min}}^l \\
P_k^l(T + 1) &= P_{\text{max}}, \quad \text{if } Y_k^l \left( P_k^l(T) \right) \geq P_{\text{max}}
\end{align*}
\]

(21)

where

\[
Y_k^l \left( P_k^l(T) \right) = \frac{\alpha_k}{\sum_{j \in \Omega_l, j \neq k} G_{BS[j],j}^l \frac{\alpha_j}{I_j} + \mu_{BS[j]}}
\]

(22)

and \(c_j^l(P_k^l(T))\) is the interference information of user \(j\) in subcarrier \(l\), i.e.,

\[
c_j^l(P_k^l(T)) = \frac{\alpha_j}{I_j \left( P_k^l(T) \right)}.
\]

As \(Y_k^l\) is a standard interference function [30], it can be shown, similar to [9], that the iterative process converges to a unique fixed point, which is equal to the global optimum of (19). The iterative algorithm independently performs power control on each subcarrier by taking into account the dual prices. The dual prices are then updated according to the sum power constraint. \(\kappa\) is the step for dual price evaluation.

The procedure used to solve the initial problem [see (17)] is detailed in Phase II of Algorithm 2. The convergence proof of this algorithm is similar to that used for multichannel DADP in [12]. The dual-price update can be viewed as a distributed gradient-projection algorithm to solve the master problem. This algorithm converges if \(\kappa\) is small enough. The proof of [12] remains valid even if, contrary to Huang et al., we consider several users per cell. Indeed, the users served by the same BS are only differentiated by their channel gains, and the sum power constraint applies to the whole cell.

The complexity of the power control algorithm is polynomial in \(O(L_{SC}(N_{BS})^2)\).

Algorithm 2 Distributed power control

(A) Phase I

for \(l = 1\) to \(L_{SC}\) do

Initialization \((T = 0) : \forall k \in \Omega_l, P_k^l(0) = P_{\text{max}}\). Compute \(\zeta_k^l(P_k^l(0))\) with (15).

while Power values have not converged do

(a) \(\text{Power update: } \forall k \in \Omega_l, \text{ compute } P_k^l(T + 1)\) with (13).

(b) \(\text{Interference information update: } \forall k \in \Omega_l, \text{ compute } c_j^l(P_k^l(T + 1))\) with (15).

(c) \(T = T + 1\).

end while

end for

(B) High weighted SINR condition

for \(l = 1\) to \(L_{SC}\) do

\(\forall k \in \Omega_l, \text{ if } c_k^l < (1/(2(\beta/\alpha_k) - 1)), \text{ remove } k \) from \(\Omega_l\).

end for
D. Graph-Based Subcarrier Allocation and Power Control

Graph-based subcarrier allocation may be followed by power control. As Algorithm 1 only allows simultaneous transmission of users that are not highly interfering with each other, we assume that the high weighted SINR condition [see (16)] is always fulfilled. Consequently, power control is then reduced to Phase II. The complexity is in $O((N_{BS}K)^3)$.

V. SIMULATION RESULTS

The simulations aim at comparing four algorithms: Algorithm 1 followed by Phase II of Algorithm 2 (Graph-Based SC + PC) or by equal power allocation (Graph-Based SC + equal power allocation (EPA)) and distributed subcarrier allocation followed by Algorithm 2 (2Phase PC) or by binary power allocation (Binary PA).

EPA provides an equal share of $P_{\text{max}}$ to each active subcarrier. Distributed subcarrier allocation consists of independently allocating subcarriers of each BS to the users that maximize the weighted rate $w_k \log_2(1 + \text{SNR}_k^l)$ with $\text{SNR}_k^l = C_{\text{BS}[k],l} P_{\text{max}}/N_0 L_{\text{SC}}$. Finally, the Binary PA algorithm [7] independently selects on each subcarrier $l \in \{1, \ldots, L_{\text{SC}}\}$ the binary combination of powers $p_{n_{\text{BS}}}(0, L_{\text{SC}})$ for all BS $n_{\text{BS}} \in \{1, \ldots, N_{\text{BS}}\}$, which maximizes the weighted sum throughput. The complexity of this algorithm is exponential in $N_{\text{BS}} : O(L_{\text{SC}} N_{\text{BS}}^3)$.

The following simulation parameters are used: The path loss model is Okumura–Hata [31]: $l(d) = 137.74 + 35.22 \log(d)$ in decibels, the shadowing’s standard deviation is 7 dB, and the thermal noise spectral density is $N_0 = -174$ dBm/Hz. The maximum transmit power for each BS is $P_{\text{max}} = 43$ dBm, and the total bandwidth is $B = 10$ MHz. The cells have omnidirectional antennas and the same cell radius with intersite distance $d_{\text{avg}} = 0.35 \sqrt{3}$ km. The location of users is uniformly distributed within each cell. Finally, the precision for 2Phase PC is $\beta = 0.15$.

A. Preliminary Results

The preliminary results concern two interfering cells in single-carrier transmission with fixed normalized weights $w_1$ and $w_2$. Monte Carlo simulations are used to compare the four algorithms with optimal resource allocation, which consists of selecting the best solution in $P_{\text{point}}$ [see (6)]. This algorithm always reaches the global optimum of the WSTM problem. Fig. 3 represents the relative weighted sum throughput loss between optimal resource allocation and the four tested algorithms. The optimal weighted sum throughput is between 70.590 kbps for $w_1 = 0.5$ and 112.958 kbps for $w_1 = 1$ and increases with $w_1$. The relative performance loss remains below 5% with all four algorithms. Graph-Based SC + PC achieves the closest weighted sum throughput to the optimum, as the highest error is then only equal to 0.77%.

These preliminary results do not enable us to conclude on the behavior of these algorithms in multicell OFDMA. However, the almost optimal performances obtained by combining criterion (9) with Phase II of Algorithm 2 in this specific subcase are the rationale for extending this method to the general case.

B. Dynamic Simulations for Cross-Layer WSTM

We now consider a network consisting of one central cell BS$_0$ and one ring of six interfering BS$_S$ separated into two nonadjacent sets $C_1 = \{\text{BS}_1, \text{BS}_3, \text{BS}_5\}$ and $C_2 = \{\text{BS}_2, \text{BS}_4, \text{BS}_6\}$. The number of available subcarriers per BS is $L_{\text{SC}} = 256$. It is equal to the fast Fourier transform size. The location of users does not vary in a given Monte Carlo sample.

We study a dynamic scenario where problem (2) is solved in each transmit time interval (TTI). The weight of each user for a given TTI is equal to its instantaneous queue length, normalized with respect to the queue lengths of all users in the network. Each user receives packets in its queue according to a Poisson traffic model, where the interarrival law follows an exponential distribution with mean of 20 TTIs. The TTI duration is 2 ms. The packet size follows a log-normal distribution law with
mean of 2.5 kb for the users of BS₀, 5 kb for the users of C₁, and 1.25 kb for the users of C₂. The packet size standard deviation is equal to 0.1 kb. The queue length cannot exceed 1024 kb, and packets arriving in the queue when this limit is reached are discarded. In each TTI, the queue lengths are updated after data transmission. WSTM is consequently a cross-layer resource-allocation problem involving medium access control and physical layers. In each Monte Carlo sample, the dynamic test runs over 1000 TTIs. The statistics, aside from the final queue length per user on Fig. 8, are averaged over all TTIs in all Monte Carlo samples.

Fig. 4 represents the weighted sum throughput and the sum throughput aggregated on all cells. To simplify reading, the sum throughput is divided by a factor of 10. When the users’ load increases, users have a lower probability of accessing the resource and suffer from higher intercell interference. Thus, when the load passes from 16 to 36 users per cell (corresponding to a multiplication by 2.25), the sum throughput is only multiplied by 1.3. This explains why the weighted sum throughput with normalized weights decreases when the load increases. The proposed algorithms are always more efficient than Binary PA in terms of WSTM. At low loads, the intercell interference is not a limiting feature, and therefore, Graph-Based SC is too restrictive. Distributed subcarrier allocation, which favors users with high weighted SNR-based rates, is then more appropriate. At medium and high loads, however, a higher WSTM is achieved by avoiding interference-limited situations thanks to Graph-Based SC. 2Phase PC efficiently adapts the interference level, leading to high weighted sum throughputs whatever the load.

The four algorithms are compared in terms of resource consumption in Figs. 5–7. The percentage of active TTIs is equal to the ratio of TTIs where at least one user of the network is active. The influence of the traffic load is evaluated by distinguishing three BSs with different incoming packet size distributions. On cells with low and medium traffic (BS₀ and BS₂), the percentage of active TTIs is limited to 45% with Graph-Based SC, whereas it goes up to 90% with 2Phase PC and to 95% with Binary PA. Graph-Based SC, with EPA or PC, enables the users of these cells to efficiently empty their queues whenever they access the resource, whereas the two other algorithms lead to useless resource access. This is due to the interference levels: With Graph-Based SC, the average intercell interference is 20–30 dBm lower than with 2Phase PC and Binary PA. Therefore, 2Phase PC is less favorable than Graph-Based SC at high user load, although it leads to the same weighted sum throughput. PC after Graph-Based SC allows additional intercell interference limitation as well as a power decrease. In this case, the power of the users with the lowest traffic (users of BS₂) is decreased to limit the interference received by the users with the highest traffic (users of BS₁), who need higher data rates to empty their queues.

Fig. 8 represents the final queue lengths of users of BS₀, BS₁, and BS₂ after 1000 TTIs. Graph-Based SC (with EPA as well as PC) and 2Phase PC favor the users of BS₁, who have the highest incoming packet traffic, and consequently the highest
queue lengths, at the cost of an increase in the queue lengths of the other users. At low user loads, the lowest queue lengths for users of BS$_1$ are obtained with 2Phase PC, whereas at medium-to-high user loads, they are obtained with Graph-Based SC. Contrary to the proposed algorithms, Binary PA is unable to cope with the queue length increase of the users of BS$_1$.

The various simulation results reported in this section show that, when considering WSTM and fairness in queue length distribution, 2Phase PC is the most efficient algorithm at low user loads, whereas Graph-Based SC + PC becomes more efficient at medium to high loads. However, Graph-Based SC + PC has the main advantage of importantly decreasing resource consumption at any load. Consequently, this algorithm is the most appropriate for WSTM.

VI. CONCLUSION

This paper has investigated resource allocation for the WSTM problem in downlink multicell OFDMA networks. The proposed algorithms, although suboptimal, effectively address WSTM. Their main advantages, with respect to state-of-the-art algorithms, are that they only require a polynomial-time complexity and that they do not make any simplifying assumption on SINR levels. From a cross-layer perspective, the proposed algorithms balance the queue lengths of all users when the weight of each user is proportional to its queue length in WSTM. Graph-based subcarrier allocation combined with distributed power control is the most efficient algorithm in terms of weighted sum throughput and queue-length management at medium-to-high user loads and in terms of resource consumption at any load.

REFERENCES


Mylene Pischella (M’09) received the Engineer Diploma in telecommunications and the Ph.D. degree from Ecole Nationale Supérieure des Télécommunications, Paris, France, in 2002 and 2009.

From 2002 to 2009, she was a Research Engineer with France Telecom R&D. She is currently an Assistant Professor with Institut Supérieur d’Electronique de Paris. Her research interests include resource allocation and interference mitigation in multicell networks, cooperative communications and cross-layer optimization, and quality-of-service provisioning.

Jean-Claude Belfiore (M’90) was born in Nice, France, in 1962. He received the Diploma in electrical engineering from SUPELEC, Gif-sur-Yvette, France, in 1985 and the Ph.D. degree from Ecole Nationale Supérieure des Télécommunications, (ENST), Paris, France, in 1989.

From 1985 to 1989, he was with Alcatel, France. He then became an Assistant Professor with ENST, where he is currently a Professor with the Communication and Electronics Department. His current research interests are in lattice coding for fading channels, space–time coding, joint source-channel coding, and wireless communication systems.