# Channel Tracking using Particle Filtering in unresolvable Multipath Environments

Tanya Bertozzi<sup>†\*</sup>, Didier Le Ruyet<sup>\*</sup>, Cristiano Panazio<sup>\*</sup> and Han Vu-Thien <sup>\*</sup>

<sup>†</sup> DIGINEXT, 45 impasse de la Draille, 13857 Aix en Provence Cedex 3, France, Tel.: 0033 4 42 90 82 82, Fax: 0033 4 42 90 82 80, Email: bertozzi@diginext.fr

\* CNAM, 292 rue Saint Martin, 75141 Paris Cedex 3, France Tel.: 0033 1 58 80 84 91, Email: leruyet@cnam.fr

#### **Abstract**

In this paper we propose a new timing error detector for timing tracking loops inside the Rake receiver in spread spectrum systems. Based on a particle filter, this timing error detector jointly tracks the delays of each path of the frequency selective channels. Instead of using conventional channel estimator we have introduced a joint time delay and channel estimator without almost no additional computational complexity. The proposed scheme avoids the drawback of the classical early late gate detector which is not able to separate closely spaced paths. Simulation results show that the proposed detectors outperform the conventional early late gate detector in indoor scenarios.

#### **Index Terms**

Sequential Monte Carlo, multipath channels, importance sampling, timing estimation.

#### I. Introduction

In wireless communications, Direct-Sequence Spread Spectrum (DS-SS) techniques have received an increasing interest, especially for the third generation of mobile systems. In DS-SS systems, the adapted filter typically employed is the Rake receiver. This receiver is efficient to counteract the effects of frequency-selective channels. It is composed of fingers, each assigned to one of the most significant channel paths. The outputs of the fingers are combined proportionally to the power of each path for estimating the transmitted symbols (maximum ratio combining). Unfortunately, the performance of the Rake receiver strongly depends on the quality of the estimation of the parameters associated with the channel paths. As a consequence, we have to estimate the delay of each path using a Timing Error Detector (TED). This goal is generally achieved in two steps: acquisition and tracking. During the acquisition phase, the number and the delays of the most significant paths are determined. These delays are estimated within one half chip from the exact delays. Then, the tracking module refines the first estimation and follows the delay variations during the permanent phase. The conventional TED used during the tracking phase is the Early Late Gate-TED (ELG-TED) associated with each path. It is well known that the ELG-TED works very well in the case of a single fading path. However, in the presence of multipath propagation, the interference between the different paths can degrade its performance. In fact, the ELG-TED cannot separate the individual paths when they are closer than one chip period from the other paths, whereas a discrimination up to  $T_c/4$  can still increase the diversity

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of the receiver ( $T_c$  denotes the chip time) [1]. When the difference between the delays of two paths is contained in the interval 0-1.5  $T_c$ , we are in the presence of unresolvable multipaths. This scenario corresponds for example to the indoor scenario. The problem of unresolvable multipaths has been recently analyzed in [2] [3] [4].

Particle filtering (PF) or Sequential Monte Carlo (SMC) methods [5] represent the most powerful approach for the sequential estimation of the hidden state of a nonlinear dynamic model. The solution to this problem depends on the knowledge of the Posterior Probability Density (PPD) of the hidden state given the observations. Except in a few special cases including linear Gaussian system models, it is impossible to calculate analytically a sequential expression of this PPD. It is necessary to adopt numerical approximations. The PF methods give a discrete approximation of the PPD of the hidden state by weighted points or particles which can be recursively updated as new observations become available.

The first main application of the PF methods was target tracking. More recently, these techniques have been successfully applied in communications, including blind equalization in Gaussian [6] and non-Gaussian [7], [8] noises and joint symbol and timing estimation [9]. For a complete survey of the communication problems dealt with using PF methods, see [10].

In this paper we propose to use the PF methods for estimating the delays of the paths in multipath fading channels. Since these methods are based on a joint approach, they provide optimal estimates of the different channel delays. In this way, we can overcome the problem of the adjacent paths which causes the failure of the conventional single path tracking approaches in the presence of unresolvable multipaths. Moreover, we will combine the PF-based TED (PF-TED) with a conventional estimator for estimating the amplitudes of the channel coefficients. We will also apply the PF methods to the estimation of the channel coefficients in order to jointly estimate the delays and the coefficients.

This paper is organized as follows. In Section 2, we will introduce the system model. Then in Section 3, we will describe the conventional ELG-TED and the PF-TED. In Section 4, we will present the conventional estimators of the channel coefficients and the application of the PF methods to the joint estimation of the delays and the channel coefficients. In Section 5, we will give simulation results. Finally, we will draw a conclusion in Section 5.

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### II. SYSTEM MODEL

We consider a DS-SS system sending a complex data sequence  $\{s_n\}$ . The data symbols are spread by a spreading sequence  $\{d_m\}_{m=0}^{N_s-1}$  where  $N_s$  is the spreading factor.

The resulting baseband equivalent transmitted signal is given by:

$$e(t) = \sum_{n} s_n \sum_{m=0}^{N_s - 1} d_m g(t - mT_c - nT), \tag{1}$$

where  $T_c$  and T are respectively the chip and symbol period and g(t) is the impulse response of the root-raised cosine filter with a rolloff factor equal to 0.22 in the case of the Universal Mobile Telecommunications System (UMTS) [11].

 $h(t, \tau)$  denotes the overall impulse response of the multipath propagation channel with  $L_h$  independent paths (Wide Sense Stationary Uncorrelated Scatterers (WSSUS) model):

$$h(t,\tau) = \sum_{l=1}^{L_h} h_l(t)\delta(\tau - \tau_l(t)). \tag{2}$$

Each path is characterized by its time-varying delay  $\tau_l(t)$  and channel coefficient  $h_l(t)$ .

The signal at the output of the matched filter is given by:

$$r(t) = \sum_{l=1}^{L_h} h_l(t) \sum_n s_n \sum_{m=0}^{N_s - 1} d_m R_g(t - mT_c - nT - \tau_l(t)) + \tilde{n}(t),$$
(3)

where  $\tilde{n}(t)$  represents the Additive White Gaussian Noise (AWGN) n(t) filtered by the matched filter and

$$R_g(t) = \int_{-\infty}^{+\infty} g^*(\tau)g(t+\tau)d\tau \tag{4}$$

is the total impulse response of the transmission and receiver filters.

Fig. 1 shows the equivalent lowpass transmission model considered in this paper.

The output of the matched filter is used as the input of the Rake receiver. The Rake receiver model is shown in Fig. 2. The Rake receiver is composed of L branches corresponding to the L most significant paths. In the l-th branch, the received and filtered signal r(t) is sampled at time  $mT_c + nT + \hat{\tau}_l$  in order to compensate the timing delay  $\tau_l$  of the associated path with the estimate  $\hat{\tau}_l$ . The outputs of each branch are combined to estimate the transmitted symbols. The output of the Rake receiver is given as:

$$\hat{s}_n = \hat{s}(nT) = \frac{1}{N_s} \sum_{l=1}^L \hat{h}_l^* \sum_{m=0}^{N_s - 1} d_m^* r(mT_c + nT + \hat{\tau}_l).$$
 (5)

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# III. THE TIMING ERROR DETECTION

#### A. The conventional TED

The Rake receiver needs good timing delays and channel estimators for each path to extract the most signal power from the received signal and to maximize the signal-to-noise ratio at the output of Rake receiver.

The conventional TED for DS-SS systems is the ELG-TED. The ELG-TED is devoted to the tracking of the delay of one path. It is composed of the early and late branches. The signal r(t) is sampled at time  $mT_c + nT + \hat{\tau}_l \pm \Delta$ . In this paper, we will use  $\Delta = \frac{T_c}{2}$ . We will restrict ourselves to the coherent ELG-TED where the algorithm uses an estimation of the transmitted data or the pilots when they are available. The output of a coherent ELG-TED associated with the l-th path is given by :

$$x_n = x(nT) = \text{Re}\left\{\hat{s}_n^* h_l^* \sum_{m=nN_s}^{(n+1)N_s - 1} \left(r(mT_c + \hat{\tau}_l + T_c/2) - r(mT_c + \hat{\tau}_l - T_c/2)\right) \hat{d}_m^*\right\}$$
(6)

The main limitation of the ELG-TED is its discrimination capability. Indeed, when the paths are unresolvable (separated by less than  $T_c$ ), the ELG-TED is not able to correctly distinguish and track the path. This scenario corresponds for example to the indoor case.

These drawbacks motivated the proposed PF-TED.

# B. The PF-TED

We propose to use the PF methods in order to jointly track the delay of each individual path of the channel. We assume that the acquisition phase has allowed us to determine the number of the most significant paths and to roughly estimate their delay.

The PF methods are used to sequentially estimate time-varying quantities from measures provided by sensors. In general, the physical phenomenon is represented by a state space model composed of two equations: the first equation describes the evolution of the unknown quantities called hidden state ( $evolution\ equation$ ) and the second equation the relation between the measures called observations and the hidden state ( $observation\ equation$ ). Given the initial distribution of the hidden state, the estimation of the hidden state at time t based on the observations until time t, is known as Bayesian inference or Bayesian filtering. This estimation can be obtained through the knowledge of two distributions: the PPD of the sequence of hidden

states from time 1 to time t given the corresponding sequence of observations and the marginal distribution of the hidden state at time t given the sequence of the observations until time t. Except in a few special cases including linear Gaussian state space models, it is impossible to analytically calculate these distributions. The PF methods provide a discrete and sequential approximation of the distributions. It can be updated when a new observation is available, without reprocessing of the previous observations. The support of the distributions is discretized by particles, which are weighted samples evolving in time.

Tracking the delay of the individual channel paths can be interpreted as a Bayesian inference. The delays are the hidden state of the system and the model (3) of the received samples relating the observations to the delays represents the observation equation. We notice that this equation is nonlinear with respect to the delays and as a consequence, we cannot analytically estimate the delays. To overcome this nonlinearity, we propose to apply the PF methods.

The PF methods have been previously applied for the delay estimation in DS-CDMA systems [12], [13]. In [12], the PF methods are used to jointly estimate the data, the channel coefficients and the propagation delay. In [13], the PF methods are combined with a Kalman Filter (KF) to respectively estimate the delay propagation and the channel coefficients; the information symbols are assumed known, provided by a Rake receiver. In both papers, the delays of each channel path are considered known and multiple of the sampling time; therefore, only the propagation delay is estimated. In this paper, the approach is different. We suppose that each channel path has a slow time-varying delay, unknown at the receiver. This environment can represent an indoor wireless communication. We assume that the information symbols are known or have been estimated essentially for three reasons:

- To reduce the computational complexity of the receiver;
- The channel estimation is typically performed transmitting known pilot symbols, for example using a specific channel as the Common Pilot Channel (CPICH) of the UMTS;
- The PF methods applied to the estimation of the information symbols perform slightly worse than simple deterministic algorithms [12], [14].

Firstly, we will apply the PF methods only to the estimation of the delays of each channel path, considering that the channel coefficients are known. In the next paragraph, we will introduce the estimation of the channel coefficients.

The structure of the proposed PF-TED is shown in Fig. 3. This estimator operates on samples

from the matched filter output taken at an arbitrary sampling rate  $1/T_s$  (at least Nyquist sampling). Then, the samples are processed by means of interpolation and decimation in order to obtain intermediate samples at the chip rate  $1/T_c$ . These samples are the input of the particle filter. In order to reduce the computational complexity of the PF-TED and since the time variation of the delays are slow with respect to the symbol duration, we choose that the particle filter works at the symbol rate 1/T. Moreover, in order to exploit all the information contained in the chips of a symbol period, the equations of the PF algorithm are modified. The PF algorithm proposed in this paper is thus the adaptation of the PF methods to a DS-SS system.

Following [15], the evolution of the delays of the channel paths can be described as a first order AutoRegressive (AR) process:

$$\begin{cases}
\tau_{1,n} = \alpha_1 \tau_{1,n-1} + v_{1,n} \\
\vdots , \\
\tau_{L,n} = \alpha_L \tau_{L,n-1} + v_{L,n}
\end{cases} (7)$$

where  $\tau_{l,n}$  for  $l=1,\cdots,L$  denotes the delay of the l-th channel path at time  $n, \alpha_1,\cdots,\alpha_L$  express the possible time variation of the delays from a time to the next one, and  $v_1,\cdots,v_L$  are AWGN with zero mean and variance  $\sigma_v^2$ . Note that the time index n is an integer multiple of the symbol duration.

The estimation of the delays can be achieved using the Minimum Mean Square Error (MMSE) method or the Maximum A Posteriori (MAP) method. The MMSE solution is given by the following expectation:

$$\hat{\tau}_n = E[\tau_n | r_{1:n}],\tag{8}$$

where  $\tau_n = \{\tau_{1,n}, \cdots, \tau_{L,n}\}$  and  $r_{1:n}$  is the sequence of received samples from time 1 to n. The calculation of (8) involves the knowledge of the marginal distribution  $p(\tau_n|r_{1:n})$ . Unlike the MMSE solution that yields an estimate of the delays at each time, the MAP method provides the estimate of the hidden state sequence  $\tau_{1:n} = \{\tau_1, \cdots, \tau_n\}$ :

$$\hat{\tau}_{1:n} = \arg \max_{\tau_{1:n}} p(\tau_{1:n}|r_{1:n}). \tag{9}$$

The calculation of (9) requires the knowledge of the PPD  $p(\tau_{1:n}|r_{1:n})$ .

The simulations give similar results for the MMSE method and the MAP method. Hence, we choose to adopt the MMSE solution as in [9]. In order to obtain samples from the marginal

distribution, we use the Sequential Importance Sampling (SIS) approach [16]. Applying the definition of the expectation, (8) can be expressed as follows:

$$\hat{\tau}_n = \int \tau_n p(\tau_n | r_{1:n}) d\tau_n. \tag{10}$$

The aim of the SIS technique is to approximate the marginal distribution  $p(\tau_n|r_{1:n})$  by means of weighted particles:

$$p(\tau_n|r_{1:n}) \approx \sum_{i=1}^{N_p} \tilde{w}_n^{(i)} \delta(\tau_n - \tau_n^{(i)}),$$
 (11)

where  $N_p$  is the number of particles,  $\tilde{w}_n^{(i)}$  is the normalized importance weight at time n associated with the particle i and  $\delta(\tau_n - \tau_n^{(i)})$  denotes the Dirac delta centered in  $\tau_n = \tau_n^{(i)}$ .

The phases of the PF-TED based on the SIS approach are summarized below.

- 1. **Initialization**: In this paper, we apply the PF methods for the tracking phase, assuming that the number of the channel paths and the initial value of the delay for each path have been estimated during the acquisition phase [17]. We assume that the error on the delay estimated by the acquisition phase belongs to the interval  $(-T_c/2, T_c/2)$ . Hence, the a priori probability density  $p(\tau_0)$  can be considered uniformly distributed in  $(\hat{\tau}_0 T_c/2, \hat{\tau}_0 + T_c/2)$ , where  $\hat{\tau}_0$  is the delay provided by the acquisition phase. Note that the PF methods can be used also for the acquisition phase. However, the number of particles has to be increased, because we have no a priori information on the initial value of the delays.
- 2. Importance sampling: The time evolution of the particles is achieved with an importance sampling distribution. When  $r_n$  is observed, the particles are drawn according to the importance function. In general, the importance function is chosen to minimize the variance of the importance weights associated with each particle. In fact, it can be shown that the variance of the importance weights can only increase stochastically over time [16]. This means that, after a few iterations of the SIS algorithm, only one particle has a normalized weight almost equal to 1 and the other weights are very close to zero. Therefore, a large computational effort is devoted to updating paths without almost no contribution to the final estimate. In order to avoid this behavior, a resampling phase of the particles is inserted among the recursions of the SIS algorithm. To limit

this degeneracy phenomenon, we need to use the optimal importance function [16], given by:

$$\pi(\tau_n^{(i)}|\tau_{1:n-1}^{(i)}, r_{1:n}) = p(\tau_n^{(i)}|\tau_{n-1}^{(i)}, r_n).$$
(12)

Unfortunately, the optimal importance function can be analytically calculated only in a few cases, including the class of models represented by a Gaussian state space model with linear observation equation. In this case, the observation equation (3) is nonlinear and thus, the optimal importance function cannot be analytically determined. We can consider two solutions to this problem [16]:

- 1) the a priori importance function  $p(\tau_n^{(i)}|\tau_{n-1}^{(i)});$
- 2) an approximated expression of the optimal importance function by linearization of the observation equation about  $\tau_{l,n}^{(i)} = \alpha_l \tau_{l,n-1}^{(i)}$  for  $l = 1, \dots, L$ .

Since the second solution involves the derivative calculation of the nonlinear observation equation and hence very complex operations, we choose the a priori importance function as in [9]. Considering that the noises  $v_{l,n}$  for  $l=1,\cdots,L$  in (7) are Gaussian, the importance function for each delay l is a Gaussian distribution with mean  $\alpha_l \tau_{l,n-1}^{(i)}$  and variance  $\sigma_v^2$ .

3. Weight update: The evaluation of the importance function for each particle at time n enables the calculation of the importance weights [16]:

$$w_n^{(i)} = w_{n-1}^{(i)} \frac{p(r_n | \tau_n^{(i)}) p(\tau_n^{(i)} | \tau_{n-1}^{(i)})}{\pi(\tau_n^{(i)} | \tau_{1:n-1}^{(i)}, r_{1:n})}.$$
(13)

This expression represents the calculation of the importance weights if we only consider the samples of the received signal at the symbol rate. However, in a DS-SS system we have additional information provided by  $N_s$  samples for each symbol period due to the spreading sequence. Consequently, we modify (13) taking into account the presence of a spreading sequence. Indeed, observing that the received samples are independent, the probability density  $p(r_n|\tau_n^{(i)})$  at the symbol rate can be written as:

$$p(r_n|\tau_n^{(i)}) = \prod_{m=nN_s}^{(n+1)N_s - 1} p(r_m|\tau_n^{(i)}).$$
(14)

Considering (3) at the chip rate and recalling the assumptions of known symbols, the probability density  $p(r_m|\tau_n^{(i)})$  is Gaussian. Typically, the received sample  $r_m$  is complex. For the calculation

of the Gaussian distribution, we can write  $r_m$  as a bi-dimensional vector with components being the real part and the imaginary part of  $r_m$ . The probability density  $p(r_m|\tau_n^{(i)})$  is thus given by:

$$p(r_m|\tau_n^{(i)}) = \frac{1}{\pi\sigma_n^2} \exp\left\{-\frac{1}{\sigma_n^2} |r_m - \mu_m^{(i)}|^2\right\},\tag{15}$$

where  $\sigma_n^2$  is the variance of the AWGN  $\tilde{n}(t)$  in (3) and the mean  $\mu_m^{(i)}$  is obtained by:

$$\mu_m^{(i)} = \sum_{l=1}^{L} h_{l,n} s_n \sum_{k=m-3}^{m+3} d_k R_g(mT_c - kT_c - nT - \tau_{l,n}^{(i)}).$$
(16)

In order to reduce the computational complexity of the PF-TED, in (16) we have assumed that the contribution of the raised cosine filter  $R_g$  to the sum on the spreading sequence is limited to the previous 3 and next 3 samples. By substitution of (15) in (14), (14) becomes:

$$p(r_n|\tau_n^{(i)}) = \left(\frac{1}{\pi\sigma_n^2}\right)^{N_s} \exp\left\{-\frac{1}{\sigma_n^2} \sum_{m=nN_s}^{(n+1)N_s-1} \left|r_m - \mu_m^{(i)}\right|^2\right\}.$$
(17)

Assuming the a priori importance function, (13) yields:

$$w_n^{(i)} = w_{n-1}^{(i)} p(r_n | \tau_n^{(i)}) = w_{n-1}^{(i)} \left(\frac{1}{\pi \sigma_n^2}\right)^{N_s} \exp\left\{-\frac{1}{\sigma_n^2} \sum_{m=nN_s}^{(n+1)N_s - 1} \left|r_m - \mu_m^{(i)}\right|^2\right\}.$$
(18)

Finally, the importance weights in (18) are normalized using the following expression:

$$\tilde{w}_n^{(i)} = \frac{w_n^{(i)}}{\sum_{j=1}^{N_p} w_n^{(j)}}.$$
(19)

4. **Estimation**: By substitution of (11) into (10), we obtain at each time the MMSE estimate:

$$\hat{\tau}_n = \sum_{i=1}^{N_p} \tilde{w}_n^{(i)} \tau_n^{(i)}.$$
 (20)

5. **Resampling**: This algorithm presents a degeneracy phenomenon. After a few iterations of the algorithm, only one particle has a normalized weight almost equal to 1 and the other weights are very close to zero. This problem of the SIS method can be eliminated with a resampling of the particles. A measure of the degeneracy is the effective sample size  $N_{eff}$ , estimated by:

$$\hat{N}_{eff} = \frac{1}{\sum_{i=1}^{N_p} (\tilde{w}_n^{(i)})^2}.$$
 (21)

When  $\hat{N}_{eff}$  is below a fixed threshold  $N_{thres}$ , the particles are resampled according to the weight distribution [16]. After each resampling task, the normalized weights are initialized to  $1/N_p$ .

# IV. THE ESTIMATION OF THE CHANNEL COEFFICIENTS

# A. The conventional estimators

Channel estimation is performed using the known pilot symbols. If we suppose that the channel remains almost unchanged during the slot, the conventional estimator of the channel coefficients of the  $l^{th}$  path is obtained by correlation using the known symbols [18]:

$$\hat{h}_{l} = \frac{1}{N_{pilot}N_{s}} \sum_{n=0}^{N_{pilot}-1} \sum_{m=0}^{N_{s}-1} s_{n}^{*} d_{m}^{*} r(mT_{c} + nT + \hat{\tau}_{l,n}),$$
(22)

where  $N_{pilot}$  is the number of pilots in a slot. For each path, the received signal is sampled at time  $mT_c + nT + \hat{\tau}_{l,n}$  in order to compensate its delay. Then the samples are multiplied by the despread sequence and summed on the whole sequence of pilot symbols. The problem of this estimator is that when the delays are unresolvable, the estimation becomes biased. To eliminate this bias, we can use an estimator based on the Maximum Likelihood (ML) criterion. In [19] [1], a simplified version of the ML estimation is proposed. The channel coefficients which maximize the ML criterion are given by :

$$\hat{\mathbf{h}} = \mathbf{P}^{-1}\mathbf{a} \tag{23}$$

where  $\hat{\mathbf{h}} = (\hat{h}_1, \dots, \hat{h}_L)$ ,  $\mathbf{P}$  is a  $L \times L$  matrix with elements  $P_{ij} = R_g(\tau_{i,n} - \tau_{j,n})$  and  $\mathbf{a}$  is the vector of the channel coefficients calculated using (22).

# B. The PF-based joint estimation of the delays and the channel coefficients

We can apply the PF methods to jointly estimate the delays of each path and the channel coefficients with a very low additional cost in terms of computational complexity. This is a suboptimal solution, since the observation equation (3) is linear and Gaussian with respect to the channel coefficients. The optimal solution is represented by a Kalman Filter (KF). However, combining the PF methods and the KF to jointly estimate the delays and the channel coefficients involves the implementation of a KF. It is better to use the particles employed for the delay estimation and to associate to each particle the estimation of the channel coefficients.

In this case, the hidden state is composed of the L delays and the L channel coefficients of each individual path. When a particle evolves in time, its new position is thus determined by the evolution of the delays and the evolution of the channel coefficients. The delays evolve as described for the PF-TED. For the channel coefficients, we assume that the time variations are

slow as for example in indoor environments. Hence, the evolution of the channel coefficients can be expressed by the following first order AR model:

$$\begin{cases} h_{1,n} = \beta_1 h_{1,n-1} + z_{1,n} \\ \vdots \\ h_{L,n} = \beta_L h_{L,n-1} + z_{L,n} \end{cases}$$
(24)

where  $\beta_1, \dots, \beta_L$  describe the possible time variation of the channel coefficients from a time to the next one and  $z_1, \dots, z_L$  are AWGN with zero mean and variance  $\sigma_z^2$ . Notice that this joint estimator operates at the symbol rate as the PF-TED.

As for the delays, we only consider the MMSE method for the estimation of the channel coefficients and we use a prior importance function:

$$\pi(h_n^{(i)}|h_{1:n-1}^{(i)}, r_{1:n}) = p(h_n^{(i)}|h_{n-1}^{(i)}), \tag{25}$$

where  $h_n = \{h_{1,n}, \dots, h_{L,n}\}$ . Considering that the noises  $z_{l,n}$  for  $l = 1, \dots, L$  in (24) are Gaussian, the importance function for the channel coefficients is a Gaussian distribution with mean  $\beta_l h_{l,n-1}^{(i)}$  and variance  $\sigma_z^2$ . To determine the positions of the particles at time n from the positions at time n-1, each particle is drawn according to  $p(\tau_n^{(i)}|\tau_{n-1}^{(i)})$  and (25).

The calculation of the importance weights is very similar to the case of the PF-TED. The only difference is that the channel coefficients  $h_{l,n}$  are replaced by the support of the particles  $h_{l,n}^{(i)}$ .

#### V. SIMULATION RESULTS

In this section, we will compare the performance of the conventional ELG-TED and the PF-TED. In order to demonstrate the gain achieved using the latter, we will consider different indoor scenarios with a two Rayleigh path channel with the same average power on each path and a maximum Doppler frequency of 19 Hz corresponding to a mobile speed of 10 Km/h for a carrier frequency of 2GHz. The simulation setup is compatible with the UMTS standard. In these conditions, the time variations of the channel delays can be expressed by the model (7), with  $\alpha_1 = \cdots = \alpha_L = 0.99999$  and  $\sigma_v^2 = 10^{-5}$  [15]. Moreover, the time variations of the channel coefficients can be represented by the model (24),  $\beta_1 = \cdots = \beta_L = 0.999$  and  $\sigma_z^2 = 10^{-3}$ .

In these simulations, a CPICH is used. In each slot of CPICH, 40 pilot symbols equal to 1 are expanded into a chip level by a spreading factor of 64. The spreading sequence is a PN sequence changing at each symbol.

# A. Tracking performance

Let's assume that the channel coefficients are known to evaluate the TED's tracking capacity and the simulation time is equal to 0.333s, corresponding to 500 slots. We have firstly considered the delays of the two paths varying according to the following model:

$$\begin{cases}
\tau_{1,n} = \alpha_1 \tau_{1,n-1} + v_{1,n} \\
\tau_{2,n} = \alpha_2 \tau_{2,n-1} + v_{2,n},
\end{cases}$$
(26)

where  $\alpha_1 = \alpha_2 = 0.999$ ,  $\sigma_{v,1}^2 = \sigma_{v,2}^2 = 0.001$ ,  $\tau_{1,0} = 0$  and  $\tau_{2,0} = 1$ .

Fig. 4 shows one realization of the considered delays and the tracking performance of two ELG-TED used for the estimation of the two delays. We assume that  $E_s/N_o=10dB$ , where  $E_s$  is the energy per symbol and  $N_o$  is the unilateral spectral power density. The classical ELG-TED presents difficulties to follow the time variation of the two delays, especially when the delay separation becomes less than 1  $T_c$ .

However, it is very important for the TED to distinguish the different paths of the channel to enable the Rake receiver to exploit the diversity contained in the multipath nature of the channel. In [1], it has been shown that the gain in diversity decreases as the separation between the paths decreases. In particular, a loss of 2.5 dB in the performance of the matched filter bound for a BER equal to  $10^{-2}$ , passing from Tc to Tc/4, has been observed. Moreover, it has been noted an interesting gain in diversity if the TED distinguishes paths separated by more than Tc/4. On the other hand, it has been found that the performance of the matched filter bound for a separation of Tc/8 is very close to the one obtained with only one path. Consequently, the TED discrimination capacity has to be equal to Tc/4. Unfortunately, the ELG-TED mistakes all the paths with a delay separation less than 1 Tc. In Fig. 5, we can observe how the discrimination capacity of the TED can be improved using the PF methods.

In order to better highlight this behavior, we have fixed the delay of the first path at 0 and the delay of the second path is decreasing linearly from  $2T_c$  to 0 over a simulation time of 0.333s corresponding to 500 slots. We assume that  $E_s/N_o=10dB$ , where  $E_s$  is the energy per symbol and  $N_o$  is the unilateral spectral power density.

Firstly, we consider that the channel coefficients are known to evaluate the TED's tracking capacity. Fig. 6 gives a representative example of the evolution of the two estimated delays using two ELG-TED. As soon as the difference between the two delays is lower than 1 Tc, due to the correlation between the two paths, the estimated delays tend to oscillate around each real delays. The ELG-TED are no longer able to perform the correct tracking of the delays. On the other hand, as shown in Fig. 8 the proposed PF-TED is able to track almost perfectly the two paths. These results have been obtained using a particle filter with only 10 particles.

Then, we have introduced the estimation of the channel coefficients into the TED. Fig. 7 shows the results obtained with two ELG-TED combined with the conventional estimator based on the correlation. As soon as the difference between the two delays is lower than 1 Tc, the detectors no longer recognize the two paths: the weaker path merges with the stronger one.

In Fig. 9, the PF-TED is also associated with the conventional estimator of the channel coefficients based on the correlation. When the delay of the second path becomes less than  $1T_c$ , the channel estimator decreases its capacity to track the time variations of the channel coefficients and the PF-TED cannot track the delays of the two paths. To improve the channel estimation, we associate the PF-TED with the ML estimator, as shown in Fig. 10. In this case, the PF-TED can track the delay of the second path up to  $T_c/2$ . For smaller delays, the PF-TED continues to distinguish the two paths, but it cannot follow the time variations of the second delay. The delay of the second path remains close to the values estimated at  $T_c/2$ .

Using the PF methods to jointly estimate the delays and the channel coefficients, we can notice in Fig. 11 that the PF-TED can track the time variations of the second path. This solution implies only a low additional cost in terms of computational complexity with respect to the PF-TED, since it exploits the set of particles used for the delay estimation for the channel coefficient estimation.

# B. Mean square error of the delay estimators

In this section, we will compare the Mean Square estimation Error (MSE) estimating  $\tau_n$  of the ELG-TED and the PF-TED with the lower posterior Cramer Rao Bound (PCRB). In the Bayesian context of this paper, the PCRB [21] is more suitable than the Cramer Rao Bound [20] to evaluate the MSE of varying unknown parameters.

The PCRB for estimating  $\tau_n$  using  $r_{1:n}$  has the form :

$$E(\hat{\tau}_n - \tau_n)^2 \ge J_{n,n}^{-1} \tag{27}$$

where  $J_{n,n}$  is the right lower element of the  $n \times n$  Fisher information matrix.

In [21], the authors have shown how to evaluate recursively  $J_{n,n}$ . For our application, the nonlinear filtering system is

$$\begin{cases} \tau_{n+1} = \alpha \tau_n + v_n \\ r_n = z_n(\tau_n) + \tilde{n}_n, \end{cases}$$
 (28)

where the second relation represents the nonlinear observation equation (3) at chip rate.

Since the spreading sequence is different at each chip time, we have to evaluate  $z_n(\tau_n)$  at this rate.

From the general recursive equation given in [21] the sequence  $\{J_{n,n}\}$  can be obtained as follows:

$$J_{n+1,n+1} = \sigma_v^{-1} + E[\nabla_{\tau_{n+1}} z_{n+1}(\tau_{n+1})]^2 \sigma_n^{-1} - (\alpha \sigma_v^{-1})^2 (J_{n,n} + \alpha^2 \sigma_v^{-1})^{-1}$$
(29)

In order to calculate  $E[\nabla_{\tau_{n+1}} z_{n+1}(\tau_{n+1})]$ , we have applied a Monte Carlo evaluation. We generate M i.i.d. state trajectories of a given length  $N_t$   $\{\tau_0^i, \tau_1^i, \dots \tau_{N_t}^i\}$  with  $1 \leq i \leq M$  by simulating the system model defined in (28) starting from an initial state  $\tau_0$  drawn from the a priori probability density  $p(\tau_0)$ . For the calculation, we fixed M = 100.

In figure 12, we show the comparison of the PCRB with the MSE estimating  $\tau_n$  of the ELG-TED and PF-TED. For both algorithms, we use an uniform initial pdf  $p(\tau_0)$ . For the PF-TED, the 10 particles were initialized uniformly in the interval  $\{-T_c/2, T_c/2\}$ . The signal to noise ratio  $E_s/N_0$  was fixed to 10 dB. We can see in the figure 12 that the PF-TED outperforms the ELG-TED and reach the PCRB bound after 15 slots. The slow convergence of the ELG-TED and PF-TED compared to the PCRB can be explained since the two TED are updated at each symbol while the PCRB bound is calculated for each chip.

### C. Performance evaluation

Fig. 13 shows the bit error rates (BER) versus  $E_s/N_o$  considering a two path channel with the same average power on each path. The delays of the first and second paths were respectively

fixed at 0 and 1  $T_c$ . The same maximum Doppler frequency as above was used. The BER values have been averaged over 50000 bits.

When using two ELG-TED, except when the channel is known the performance are very poor compare to the maximum achievable performance (known delays and channel coefficients). On the other hand, the PF-TED with channel coefficients known or estimated reaches the optimal performance. We can conclude that the considered TED must be able to separate the different paths of the channel, otherwise the performance of the Rake receiver breaks down.

# VI. CONCLUSIONS

In this paper we have proposed to use the PF methods in order to track the delay of the different channel paths. We have assumed that an acquisition phase has already provided an initial estimation of these delays.

We have firstly considered that the channel coefficients are known. We have compared the tracking capacity of the conventional ELG-TED and the proposed PF-TED. We have shown that when the delays of the channel paths become very close (less than  $1T_c$ ), the ELG-TED is unable to track the time variations of the delays. However, the PF-TED continues to track the delays.

We have introduced the channel coefficient estimation to the TED. We have considered two classical methods: the estimation based on the correlation using pilot symbols and the estimation based on the ML criterion. We have shown that the ELG-TED with estimation of the channel coefficients loses the capacity to distinguish the paths when the delays are very closed. On the other hand, the PF-TED associated with the two classical channel estimator is able to separate the different paths. However, for very close delays the channel estimators prevent to the PF-TED to track the time variations of the delays. We have proposed to estimate jointly the delays and the channel coefficients using the PF methods to avoid this loss of tracking. We have found that the joint estimation enables a better tracking of the delays.

Finally, we have seen that it is very important for the Rake receiver that the TED can distinguish the different paths of the channel. We have observed that in the case of unresolvable paths, the ELG-TED confuses the paths and the performance of the Rake receiver are very poor.

As conclusion, we can say that the PF-TED based on the joint estimation of the delays and the channel coefficients can be a good substitute of the classical ELG-TED, specially for

indoor wireless communications. Moreover, the computational complexity of the PF-TED is very limited, seen that we have used only 10 particles.

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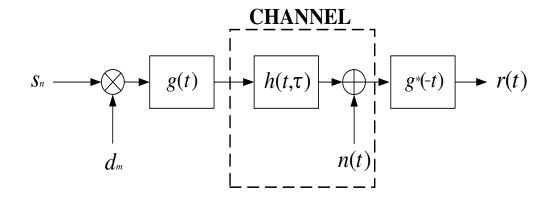


Fig. 1. Equivalent lowpass transmission system model.

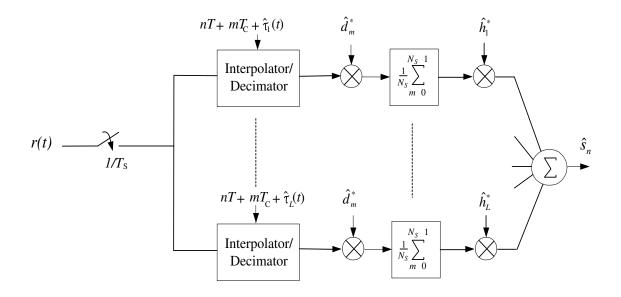


Fig. 2. Rake receiver model.

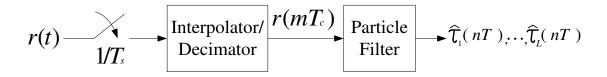


Fig. 3. Structure of the proposed PF-TED.

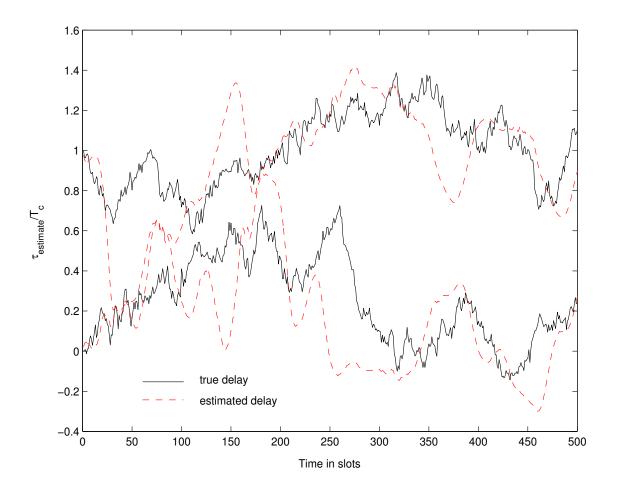


Fig. 4. Delay tracking with the conventional ELG-TED.

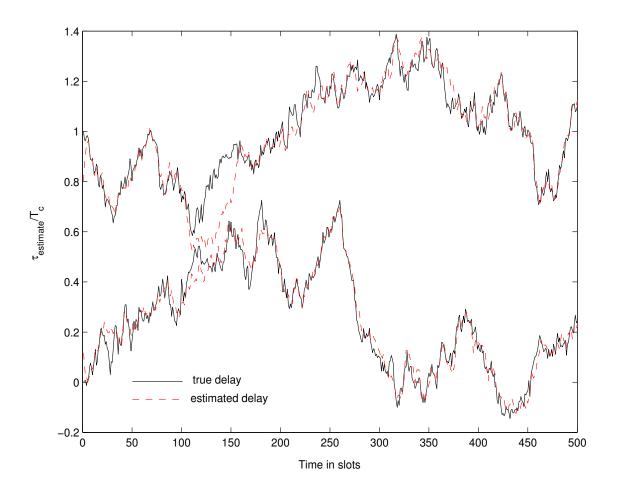


Fig. 5. Delay tracking with the PF-TED.

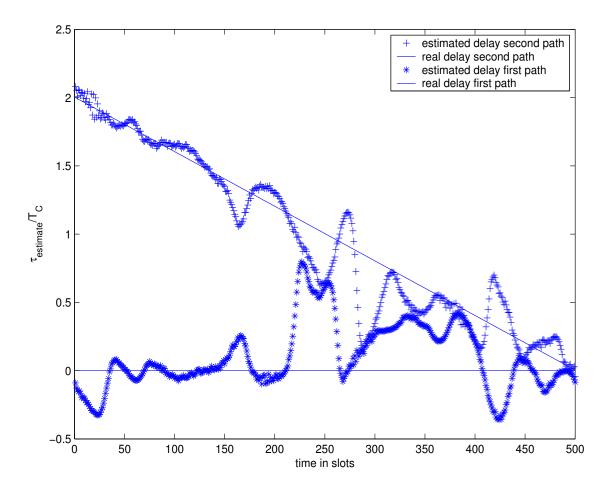


Fig. 6. Delay tracking with the conventional ELG-TED.

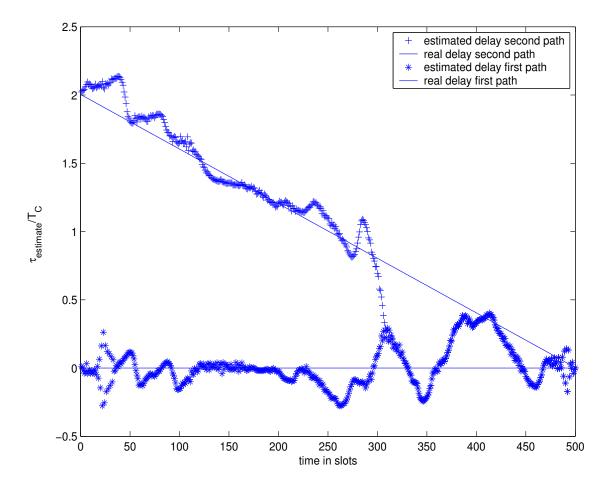


Fig. 7. Delay tracking with the conventional ELG-TED associated with a conventional channel coefficient estimator based on the correlation.

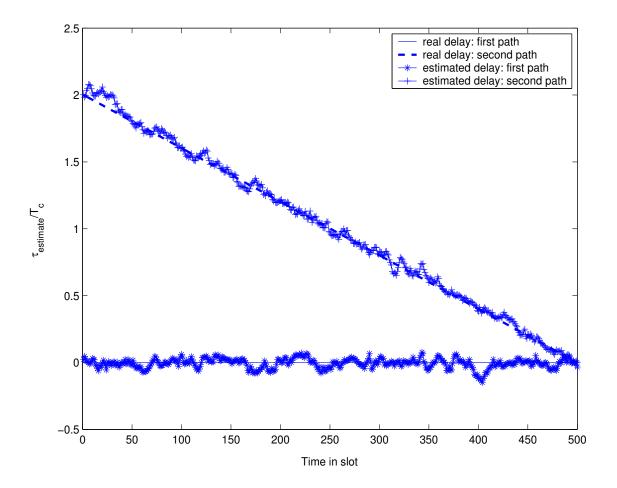


Fig. 8. Delay tracking with the PF-TED.

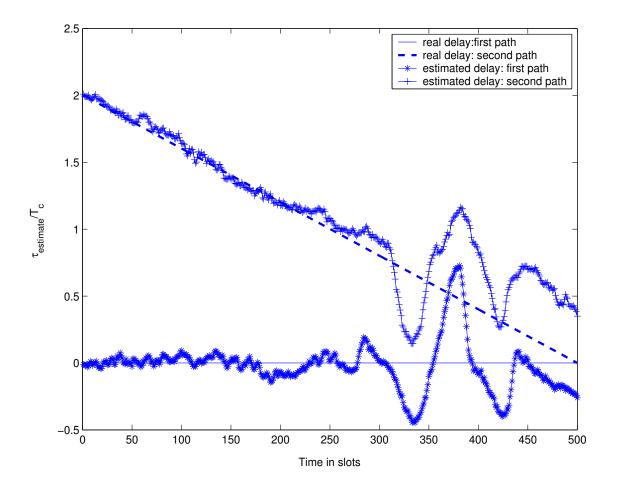


Fig. 9. Delay tracking with the PF-TED associated with a conventional channel coefficient estimator based on the correlation.

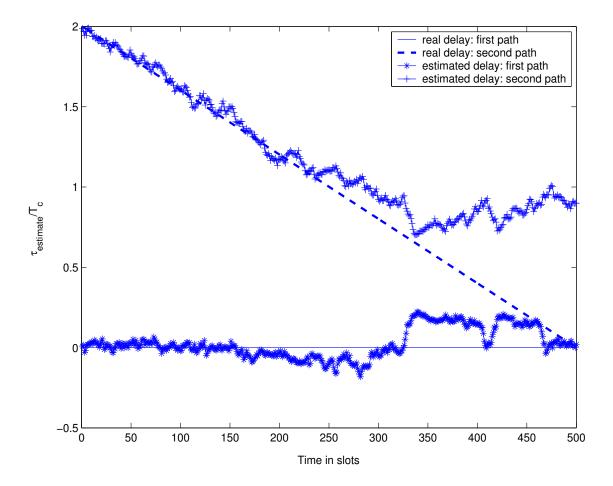


Fig. 10. Delay tracking with the PF-TED associated with a conventional channel coefficient estimator based on the ML.

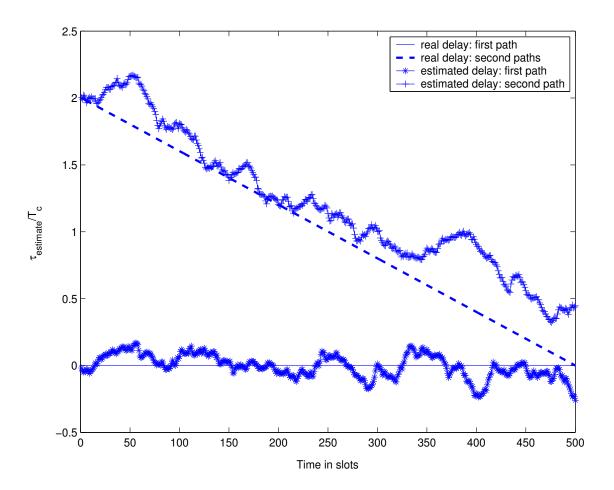


Fig. 11. Delay tracking with a joint delay and channel coefficient estimator based on the PF methods.

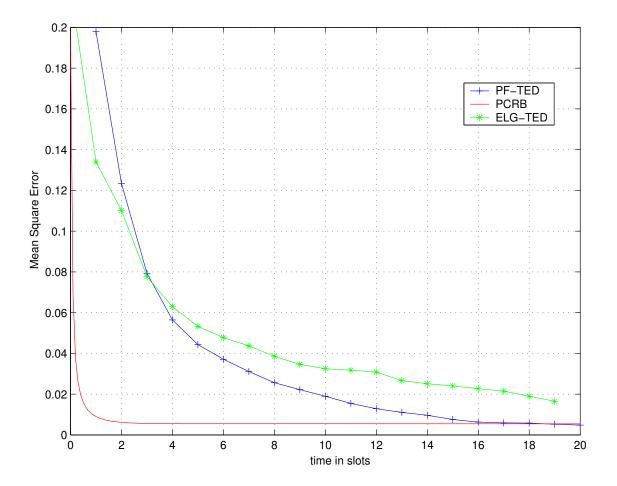


Fig. 12. Comparison of the PCRB with the MSE estimating  $au_n$  of ELG-TED and PF-TED

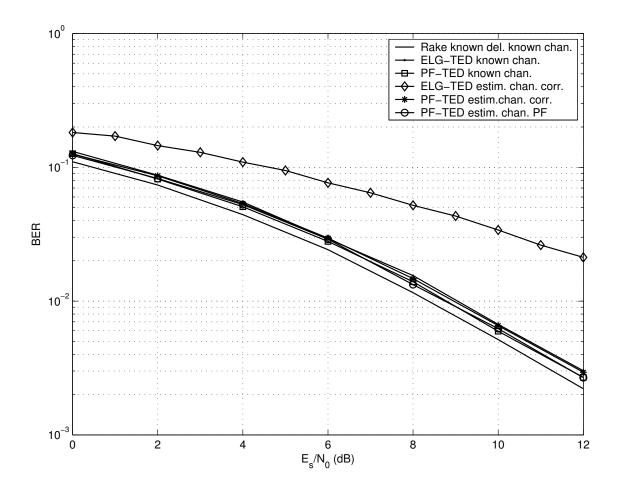


Fig. 13. Performance comparison of the ELG-TED and the PF-TED.