Adaptive Resource Allocation for Multicast OFDM Systems with Multiple Transmit Antennas

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***Abstract*— We evaluate the impact of multiple transmit antennas on the performance of multicast OFDM systems. Associated with a powerful erasure code, it is possible to increase the data rate of multicast OFDM systems by selecting for each subcarrier the users with a good channel condition. We show that the resource allocation which includes the precoding vector selection, subcarrier allocation and bit loading is a difficult optimization problem. We propose two suboptimal algorithms to solve this problem. In the first algorithm, we optimize the precoding vector using an interior point method while the second proposed algorithm avoids this optimization. When the users are not symmetrically distributed around the base station, we add a fair scheduler to guarantee that each user receives the same amount of data. We present simulation results where we compare the proposed multicast systems with classical multicast OFDM systems. When the users are symmetrically distributed around the base station, the gain is rather small, whereas when the users are non-symmetrically distributed, the second algorithm outperforms OFDM systems with subcarrier allocation.**

keywords : Multicast OFDM, multiple transmit antennas, precoding vector optimization, adaptive subcarrier allocation.

I. Introduction

Up to now, the main wireless applications are unicasting and broadcasting. However, the demand for the audio and video transmission is increasing. For these applications, multicasting offers a significant improvement compared to broadcasting. It allows the transmission of packets to multiple destinations using less resources [1]. However, since in wireless channel the received signal-to-noise ratio (SNR) of each user is not the same, the data rate of the multicast stream is limited by the data rate of the least capable user. Consequently, this method cannot provide efficient performance when the number of users in the group increases.

The main difficulty in achieving high data rate on the wireless channels is known to be frequency selectivity and the fading due to the existence of multiple paths. Orthogonal frequency division multiplexing (OFDM) techniques can significantly alleviate the impacts of frequency selective fading [2] and is attractive for the next generation of wireless systems. Moreover, multiple-input multiple output (MIMO) systems that use multiple antenna techniques [3] are combined with OFDM systems to enhance the performance of wireless systems in fading channels.

In [4], by assuming that the transmitter knows the instantaneous channel transfer function of all the users, it has been shown that adaptive subcarrier and bit allocation can significantly increase the data rate of broadcast OFDM scheme. Recently it has been demonstrated in [5] that using OFDM with subcarrier and bit allocation, it is also possible to increase the data rate on each subcarrier for multicasting applications by selecting only the users with a good channel condition.

In this paper, we propose to evaluate the impact of multiple transmit antennas on the performance of multicast OFDM systems. Our purpose is to maximize the sum rate to increase the quantity of received data by all users for a given time or to reduce the transmission delay for a given frame. We show the determination of the precoding vector, the allocation of the subcarriers to the users and the loading bits to the subcarriers is an optimization problem. We propose two suboptimal algorithms to solve this complex problem. With these algorithms, we allow one subcarrier to carry data for more than one user by optimizing the precoding vector. This approach can be combined with a fair scheduler and a powerful erasure code such as digital fountain codes [6].

The basic principle behind the use of erasure codes is that the original source data is encoded in the form of a sequence of packets. There packets are transmitted to the users. A receiver can reconstruct the original source data once it receives a sufficient number of encoded packets. The main benefit of this approach is that different receivers can recover the source data using different encoded packets. Therefore, our purpose is to maximize the total number of received packets using precoding vector optimization, subcarrier allocation and bit loading while providing the reception of almost the same amount of packets to each user using a fair scheduler.

This paper is organized as follows. First, we describe the system model of multicast OFDM systems with multiple transmit antennas over wireless channels in section II. Then, we propose two precoding vector optimization and subcarrier allocation algorithms including fair scheduler in section III. Finally, we give simulation and comparison results in section IV.

II. System model for Multicast Ofdm with multiple transmit antennas

We consider the multicast OFDM system where the base station with N_t transmit antennas serves K users with a single receive antenna, as shown in Figure 1. The common data is formed into the OFDM symbol with N subcarriers and then transmitted from N_t antennas through the frequency selective channels. For each pth OFDM frame, this channel that is between the jth transmit antenna and kth user is described using a baseband equivalent impulse response as

$$\mathbf{h}_{p,k,j} = \left[\begin{array}{cccc} h_{p,k,1,j} & h_{p,k,2,j} & \dots & h_{p,k,L_f,j} \end{array} \right]^T \quad (1)$$

where L_f is the length of the channel response. We assume that the channel is constant over one OFDM frame and that the channel taps is equal or smaller than the length of the guard interval (GI) in order to avoid intercarrier and intersymbol interference at the receiver.

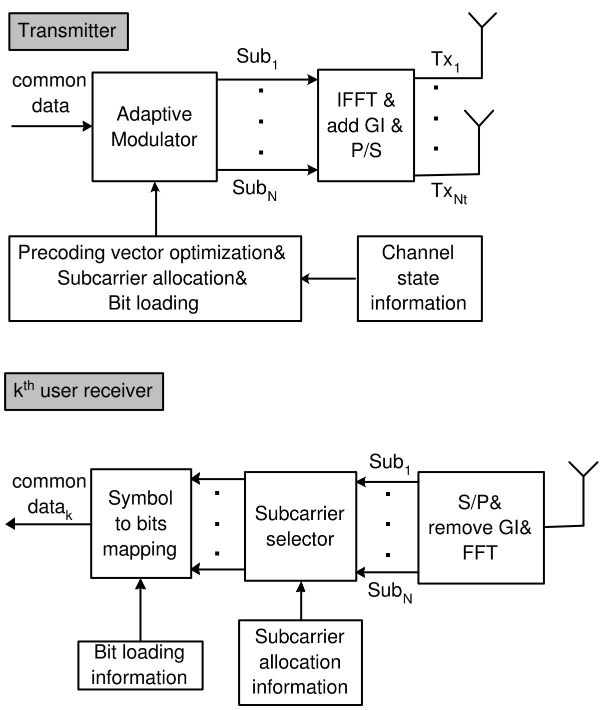

Fig. 1. The transmitter and kth receiver structure for multicast OFDM system with multiple transmit antennas

Equation (1) is written in the frequency domain as the channel vector for kth user and nth subcarrier

$$\mathbf{H}_{p,k,n} = \left[\begin{array}{cccc} H_{p,k,n,1} & H_{p,k,n,2} & \dots & H_{p,k,n,N_t} \end{array} \right]^T \quad (2)$$

where $H_{p,k,n,j}$ is the channel gain from jth transmit antenna to kth user for nth subcarrier.

Assuming that the channel information about all the subcarriers for all users are known at the transmitter, the adaptive resource allocation should optimize the precoding vector, allocate the users to the subcarrier and load bits in a way that maximizes the total number of bits received by all the users. After that, the multicast data are fed into the adaptive modulator that assigns each subcarrier to a group of users which receive the same data, and determines the number of bits on each subcarrier considering the lowest one among the channel gains of all the users allocated to this subcarrier. We also assume that the subcarrier/bit allocation information is transmitted to each user through a separate control channel.

In order to define the optimization problem, first we will define the notations. Let $R_{p,k}$ be the data rate of the kth user and $c_{p,n}$ be the number of bits that are assigned to the nth subcarrier. Here, the user index is unnecessary because the users using the subcarrier receive identical data using the same modulation. c_n is selected from the set of $\{0, 1, 2, ..., M\}$ where M is the maximum number of bits/symbol that can be transmitted by each subcarrier $n = 1, 2, ..., N$. The data rate $R_{p,k}$ can be expressed as

$$R_{p,k} = \sum_{n=1}^{N} c_{p,n}\rho_{p,k,n} \quad (3)$$

where $\rho_{p,k,n}$ is a binary value indicating whether kth user utilizes the nth subcarrier or not.

$$\rho_{p,k,n} = \begin{cases} 1 & \text{if the nth subcarrier is used for kth user} \\ 0 & \text{else} \end{cases} \quad (4)$$

Assuming that available total transmit power is limited by P_T, in order to maximize the total rate by all users, the optimization problem can be expressed as

$$\max_{\mathbf{W}_p;c_p;\rho_p} \sum_{k=1}^{K} R_{p,k} = \max_{\mathbf{W}_p;c_p;\rho_p} \sum_{k=1}^{K}\sum_{n=1}^{N} c_{p,n}\rho_{p,k,n} \quad (5)$$

subject to

$$\sum_{n=1}^{N} \max_k \left(\frac{f(c)\rho_{p,k,n}}{\|\mathbf{W}_p\mathbf{H}_{p,k,n}\|^2} \right) \leq P_T \quad (6)$$

where $\mathbf{W}_p$ is the $N_t \times 1$ precoding vector as described in Figure 2 and $f(c)$ is the required received power for reliable reception of c bits when the channel gain is one. $f(c)$ can be represented for M-ary quadrature amplitude modulation (M-QAM) by [7]

$$f(c) = \frac{N_0}{3}[Q^{-1}(p_e/4)]^2(2^c - 1) \quad (7)$$

where p_e is the required bit-error-rate (BER), $N_0/2$ denotes the variance of the additive white Gaussian noise (AWGN) and

$$Q(x) = \int_x^{\infty} \mathrm{e}^{-t^2/2}dt \quad (8)$$

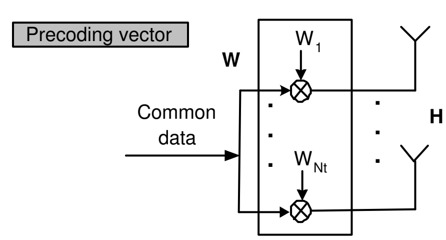

Fig. 2. The block diagram of the precoding vector for OFDM with multiple transmit antennas

III. Proposed Adaptive Resource Allocation Algorithms

The problem includes both precoding vector optimization, subcarrier allocation and bit loading. Since, it is very difficult to solve (5) subject to (6), we propose to separate the bit allocation problem and solve the precoding vector optimization and subcarrier allocation by assuming equal power for each subcarrier. Therefore, the user data rate should be maximized for each subcarrier. Then, a new suboptimal optimization expression can be written as

For each subcarrier n:

$$\max_{\mathbf{w}_{p,n};c_{p,n};\rho_{p,n}} \sum_{k=1}^{K} c_{p,n}\rho_{p,k,n} \tag{9}$$

subject to

$$\max_k \left(\frac{f(c)\rho_{p,k,n}}{\|\mathbf{W}_{p,n}\mathbf{H}_{p,k,n}\|^2} \right) \leq \frac{P_T}{N} \tag{10}$$

Except for small value of K, the optimization problem is still intractable. As a consequence, we propose two suboptimal algorithms to optimize the precoding vector and to choose the configuration of user allocation.

A. Proposed Algorithm-1

In this algorithm, first we optimize the precoding vector in order to maximize the data rate of the least capable user in the group by taking into account the power constraint. Then, we perform the subcarrier allocation to maximize the sum rate [5].

For each subcarrier n:

Step 1) Optimization of the precoding vector (max-min algorithm):

$$\max_{\mathbf{W}_{p,n}} \quad \min\{\mathbf{H}^H_{p,k,n}\mathbf{W}^H_{p,n}\mathbf{W}_{p,n}\mathbf{H}_{p,k,n}; k=1,2,...K)\} \tag{11}$$

subject to

$$\mathbf{W}_{p,n}\mathbf{W}^H_{p,n} \leqslant \frac{P_T}{N} \tag{12}$$

Step 2) Calculation of SNRs:

$$\text{SNR}_{p,k,n} = \|\mathbf{W}_{p,n}\mathbf{H}_{p,k,n}\|^2 \tag{13}$$

where $k = 1, 2, ..., K$.

Let $c_{p,k,n}$ be the number of bits that can be received by the kth user at the nth subcarrier in the case of $\rho_{p,k,n} = 1$. The number of bits for the kth user is given by

$$c_{p,k,n} = \min\Big(f^{-1}(\text{SNR}_{p,k,n}), M \Big) \tag{14}$$

where $f^{-1}(.)$ is the inverse function of $f(.)$ defined in (7) and M is the largest modulation index.

Step 3) Selection of the user that maximizes the sum rate:

Calculate the tentative total data rate $\bar{R}_{p,k,n}$ when the kth user is selected as the user requiring the maximum power.

$$\bar{R}_{p,k,n} = u_{p,k,n}c_{p,k,n} \tag{15}$$

where $u_{p,k,n}$ indicates the number of users who have the SNR larger than $\text{SNR}_{p,k,n}$. Select the user index κ maximizing $\bar{R}_{p,k,n}$.

$$\kappa = \max_k \bar{R}_{p,k,n} \tag{16}$$

Step 4) Allocation of the users to the subcarrier:

Then, the subcarrier allocation is completed by selecting the users as

$$\rho_{p,k,n} = \begin{cases} 1 & \text{if} \quad \text{SNR}_{p,k,n} \geq \text{SNR}_{p,\kappa,n} \\ 0 & \text{else} \end{cases} \tag{17}$$

While the max-min algorithm is NP-hard, using relaxation, it is possible to transform it into a form suitable for semi definite programming problem [8]. Consequently, using interior point method we are able to solve this problem.

B. Proposed Algorithm-2

In order to reduce the complexity of the *algorithm-1*, we avoid the optimization by choosing the precoding vector as one of the transpose-conjugate of channel vector of the users which maximizes the sum rate with performing the subcarrier allocation.

For each subcarrier n:

Step 1) Initializing of the precoding vector:

$$\mathbf{W}^{\text{init}}_{p,k',n} = P^{\text{init}} \left[H^*_{p,k',n,1} \quad H^*_{p,k',n,2} \quad ... \quad H^*_{p,k',n,N_t} \right] \tag{18}$$

where $P^{\text{init}} = \frac{P_T/N}{\sqrt{|H_{p,k',n,1}|^2 + ...|H_{p,k',n,N_t}|^2}}$ and $k' = 1, 2, ..., K$.

Step 2) Calculation of the SNRs:

Then, the SNR of each user for each initial precoding vector are calculated as

$$\text{SNR}_{p,k,k',n} = \|\mathbf{W}^{\text{init}}_{p,k',n}\mathbf{H}_{p,k,n}\|^2 \tag{19}$$

Let $c_{p,k,k',n}$ be number of bits that can be received by kth user using k'th initial precoding vector in the case of $\rho_{p,k,n} = 1$. The number of bits for the kth user is given by

$$c_{p,k,k',n} = \min\Big(f^{-1}(\text{SNR}_{p,k,k',n}), M \Big) \tag{20}$$

Step 3) Selection of the precoding vector and the user that maximizes sum rate:

Calculate the tentative total data rate $\bar{R}_{p,k,k',n}$ when the kth user is selected as the user requiring the maximum power for each initial precoding vector.

$$\bar{R}_{p,k,k',n} = u_{p,k,k',n}c_{p,k,k',n} \tag{21}$$

where $u_{p,k,k',n}$ indicates the number of users who have channel gains larger than $\text{SNR}_{p,k,k',n}$. Select the user index κ and the precoding vector index ϱ maximizing $\bar{R}_{p,k,k',n}$.

$$\kappa, \varrho = \max_{k,k'} \bar{R}_{p,k,k',n} \tag{22}$$

Then, the precoding vector is chosen as

$$\mathbf{W}_{p,n} = \mathbf{W}^{\text{init}}_{p,\kappa,n} \tag{23}$$

Step 4) Allocation of the users to the subcarrier:

$$\rho_{p,k,n} = \begin{cases} 1 & \text{if} \quad \text{SNR}_{p,k,\varrho,n} \geq \text{SNR}_{p,\kappa,\varrho,n} \\ 0 & \text{else} \end{cases} \tag{24}$$

C. Bit Loading Algorithm

The bit loading algorithm is considered using the modified Levin-Campello algorithm [4] [5] under the assumption that the subcarrier allocation is completed.

Let $\Delta P_{p,n}(c)$ denote the incremental power needed for the transmission of one additional bit at the subcarrier n. When the number of loaded bits for the nth subcarrier is c, $\Delta P_{p,n}(c)$ is given for the *algorithm-1*

$$\Delta P_{p,n}(c) = \frac{P_T}{N} \frac{f(c+1) - f(c)}{u_{p,n}\text{SNR}_{p,\kappa,n}} \tag{25}$$

where $u_{p,n}$ is the number of users who share the nth subcarrier, which is necessary because the incremental power is shared by the group of users allocated to the subcarrier.

$$u_{p,n} = \sum_{k=1}^{K} \rho_{p,k,n} \tag{26}$$

Then, the bit loading algorithm is summarized as follows:

Step 1) Initialization:
$c_{p,n} = 0$ and evaluate $\Delta P_{p,n}(c = 0)$ for all n.
Tentative transmit power is $P_T^* = 0$.

Step 2) Bit Loading Iteration:
repeat the following unless $P_T^* \geq P_T$
$n^* = \min_n \Delta P_{p,n}(c_{p,n})$
$P_T^* = P_T^* + \Delta P_{p,n^*}(c_{p,n^*}) u_{p,n^*}$
$c_{p,n^*} = c_{p,n^*} + 1$
if $c_{p,n^*} = M$, set $\Delta P_{p,n^*}(c_{p,n^*}) = \infty$
else evaluate $\Delta P_{p,n^*}(c_{p,n^*})$.

D. Proposed Fair Scheduling Algorithm

In order to provide that each user receives almost the same amount of packets at the end of transmission in the case of non-symmetric user distribution around the base station, we should introduce a fair scheduling algorithm. Therefore, for each user we define a priority factor that represents the ratio between the received bits by that user and the total transmitted bits. The priority factor is inversely proportional to the total received number of bits by the user.

$$P_{p,k} = \frac{\sum_{p'=1}^{p-1} \sum_{x=1}^{K} R_{p-1,x}}{R_{p-1,k}} \tag{27}$$

where $p-1$ is the index of total transmitted OFDM frame. The priority vector is reconstructed from the priority factors of all users after normalization and fourth powered.

$$\mathbf{P}_p = \left(\frac{1}{\max_k P_{p,k}} \left[\begin{array}{cccc} P_{p,1} & P_{p,2} & \dots & P_{p,K} \end{array} \right] \right)^4 \tag{28}$$

For the *algorithm-1*, the *step 3* is changed to include the priority factor as

Step 3) Selection of the user that maximizes the sum rate:

Calculate the rational total rate $\bar{O}_{k,n}$ when the kth user is selected as the user requiring maximum power.

$$\bar{O}_{p,k,n} = \mathbf{P}_p \mathbf{G}_{p,k,n} c_{p,k,n} \tag{29}$$

where $\mathbf{G}_{p,k,n} = [\rho_{p,1,n} \quad \dots \quad \rho_{p,K,n}]$ and

$$\rho_{p,x,n} = \begin{cases} 1 & \text{if} \quad \text{SNR}_{p,x,n} \geq \text{SNR}_{p,k,n} \\ 0 & \text{else} \end{cases} \tag{30}$$

Then, the selection of the user index is performed by maximizing $\bar{O}_{k,n}$.

$$\varrho^f = \max_k \bar{O}_{p,k,n} \tag{31}$$

The same derivation can also be performed for the *algorithm-2* except that we have to select both the user and precoding vector.

In order to evaluate the fairness of each algorithm, we consider the fairness index (FI) given by [9]

$$\text{FI} = \frac{\left(\sum_{k=1}^{K} R_k \right)^2}{K \sum_{k=1}^{K} R_k^2} \tag{32}$$

where $R_k = \sum_{p=1}^{P} R_{p,k}$ and P is the number of total OFDM frame. The FI ranges between 0 (no fairness) and 1 (perfect fairness).

IV. Numerical Results

In this section, we evaluate the sum data rates of the multicast OFDM systems with multiple transmit antennas using the proposed algorithms and compare them with those of the multicast OFDM systems.

The simulation results are performed using Hiperlan/2 standard [10] using channel model A [11] which corresponds to a typical office environment with 9 channel taps. The total OFDM symbol duration is $4\mu s$ including $0.8\mu s$ guard interval. It consists of $N = 64$ subcarriers. The required BER, the noise variance and the maximum number of bits per symbol are chosen as $P_e = 10^{-4}$, $N_0 = 2$ and $M = 8$ respectively. The transmitted power is fixed at $P_T = 30$dB.

In Figure 3, we show the performance results using two transmit antennas and assuming that the users are symmetrically distributed around the base station without path loss. The results indicate that the sum data rate is increased by 15% using the *algorithm-2* compared to OFDM systems with subcarrier allocation [5]. Furthermore, the max-min worst user algorithm outperforms the OFDM worst user algorithm.

In Figure 4, we draw the simulation results using two transmit antennas and assuming that the users are non-symmetrically distributed around the base station and the path loss of users is chosen from the set $[0\text{dB}, 5\text{dB}, 10\text{dB}, 15\text{dB}]$ with the ratio of 25%. In this scheme, a fair scheduler should be added to guaranty that each user receives the same amount of data. The results show that the *algorithm-2* with a fairness index close to 1 outperforms all the other algorithms. The fair scheduler reduces strongly the performance of the *algorithm-1* since the precoding vector is not modify according to the priority factors.

In Figure 5, we illustrate the impact of the number of transmit antennas on the performance of the different algorithms for the non-symmetric distributed users case. According to the results, the sum data rate remains almost constant except when using the max-min worst user algorithm. However, even with 8 transmit antennas, the *algorithm-2* is more suitable than the max-min worst user in practice.

V. Conclusion

In this paper, we have evaluated the impact of multiple transmit antennas on the performance of multicast OFDM systems. We have shown that the resource allocation which includes the precoding vector selection, subcarrier allocation and bit loading is a difficult optimization problem. We have proposed two suboptimal algorithms for the resource allocation. In the first algorithm, we have optimized the precoding vector using an interior point method whereas the second proposed algorithm have avoided this optimization. Compared to multicast OFDM systems, we have shown that it is possible to increase the sum data rate with multiple transmit antennas. When the users are symmetrically distributed around the base station, the gain is rather small (15%); on the other hand, when the users are non-symmetrically distributed, we have shown that the *algorithm-2* outperforms OFDM systems with subcarrier allocation.

We have also obtained better performance with the *algorithm-2* compared to the max-min worst user for both symmetrically and non-symmetrically user distribution in the case of two transmit antennas without performing optimization. However, for more than 4 transmit antennas, the achievable gain of the max-min worst user is higher than the *algorithm-2*.

It is of interest to extend these results by evaluating the impact of the feedback and quantization errors on the performance of the proposed algorithms.

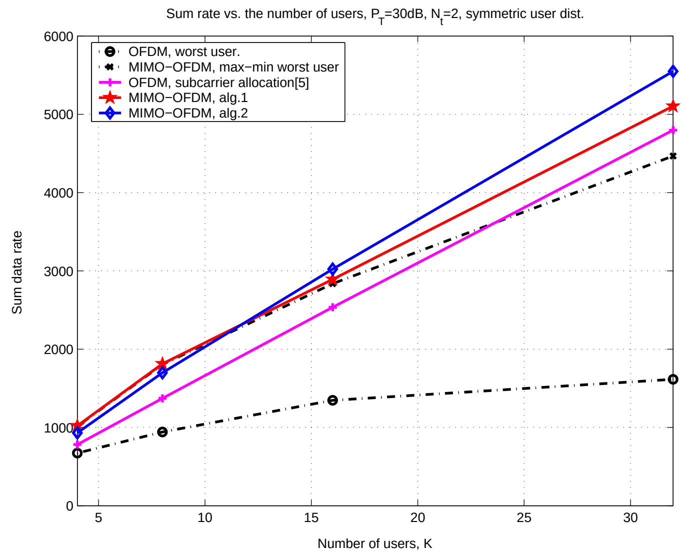

Fig. 3. Sum rate versus the number of users for $P_T = 30$dB and $N_t = 2$ for symmetric user distribution

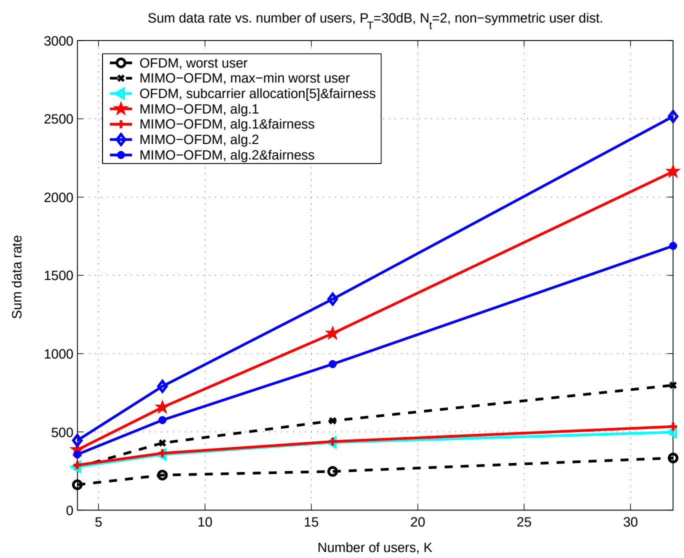

Fig. 4. Sum rate versus the number of users for $P_T = 30$dB and $N_t = 2$ for non-symmetric user distribution

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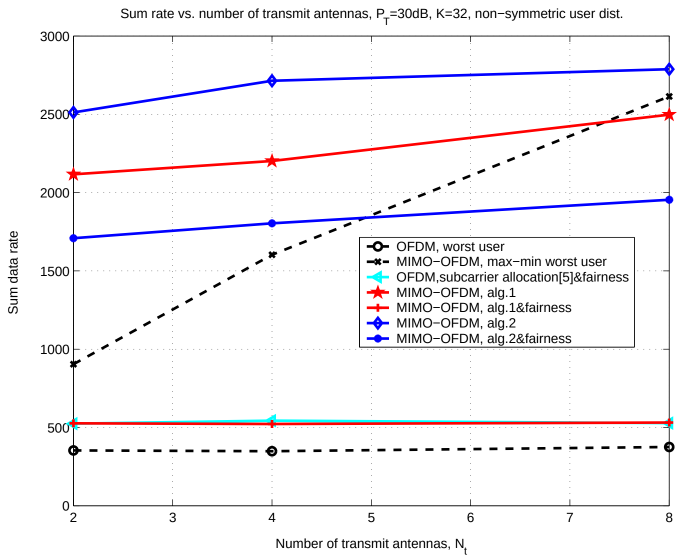

Fig. 5. Sum rate versus the number of transmit antennas for $P_T = 30$dB and $K = 32$ for non-symmetric user distribution

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